UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE
DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

MAIN EXAMINATION 2011/12
TITLE OF PAPER: SOLID STATE ELECTRONICS
COURSE NUMBER: EE429
TIME ALLOWED: 3 HOURS

INSTRUCTIONS:
ANSWER ANY FOUR OUT OF FIVE QUESTIONS.
EACH QUESTION CARRIES 25 MARKS.
MARKS FOR DIFFERENT SECTIONS ARE SHOWN ENCLOSED IN SQUARE BRAKETS.

THIS PAPER HAS 5 PAGES INCLUDING THIS PAGE.
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1. (a) The primitive translation vectors of the hexagonal space lattice may be taken as $\boldsymbol{a}=\left(3^{1 / 2} a / 2\right) i+(a / 2) j ; \quad b=-\left(3^{1 / 2} a / 2\right) i+(a / 2) j ; \quad c=c k$. Show that the volume of the primitive cell is $\left(3^{1 / 2} / 2\right) a^{2} c$.
(b) Show that for a cubic lattice, the lattice constant $a$ is given by:

$$
a=\left[\frac{n M_{A} \rho}{N_{A} \rho}\right]^{1 / 3}
$$

where $n$ is the number of atoms per unit cell, $M_{A}$ is the atomic mass, $N_{A}$ is the Avogadro's number and $\rho$ is the density of the crystalline material.
(c) (i) Find the atomic density (not mass density) of crystalline Si on the (110) plane.
(ii) Find the distance between two adjacent (111) planes in crystalline Si. The lattice constant for Si is: $a=5.43 \times 10^{-8} \mathrm{~cm}$.
(d) Most semiconductors crystallize in a diamond or the related zinc blend structure. Find the maximum packing fraction of identical spheres in a diamond crystal structure.
2. (a) A Ge crystal is to be grown by the Czochralski method and it is desired that the ingot contains $10^{14}$ atoms $/ \mathrm{cm}^{3}$ of arsenic (As) as a dopant.
(i) Calculate the density of Ge from its lattice constant, $a=5.65 \times 10^{-8} \mathrm{~cm}$. The atomic weight of Ge is 72.6 .
(ii) What concentration of As atoms should the melt contain to give the impurity concentration of $10^{14}$ atoms $/ \mathrm{cm}^{3}$ in the resulting crystal? Use the distribution coefficient, $k_{d}=0.4$.
(iii) If the initial load of Ge in the crucible is 4 kg , how many grams of As should be added? The atomic weight of As is 74.9.
(b) The energy of the $n^{\text {th }}$ Bohr orbit of the hydrogen atom is given by:

$$
E_{n}=-\frac{k e^{2}}{2 a_{0}}\left(\frac{1}{n^{2}}\right)
$$

where all symbols have their usual meaning. Consider a hydrogen-like beryllium ion $\left(\mathrm{Be}^{3+}\right)$ which has four protons in its nucleus (atomic number $Z=4$ ) and one electron in orbit.
(i) Use the Bohr model of the hydrogen atom to calculate the energies of the first four electronic states in the ion. Comment on the energy gaps between successive states.
(ii) Find the ionization energy for the removal of the sole electron from the ion.
(c) A particle in an infinite potential well from $x=0$ to $x=L$ in the $n=1$ state has the wave function:

$$
\psi(x)=\sqrt{\frac{2}{L}} \sin \left(\frac{\pi x}{L}\right) .
$$

Find an expression for the probability of finding the particle anywhere form $x=0$ to $\mathrm{x}=l$, where $0 \leq l \leq L$.
3. (a) The respective concentrations of electrons and holes in the conduction band of a semiconductor can be given by:

$$
\begin{aligned}
& n=2\left(\frac{2 \pi m_{e} k_{B} T}{h^{2}}\right)^{3 / 2} e^{\left(E_{F}-E_{C}\right) / k_{B} T} \\
& p=2\left(\frac{2 \pi m_{h} k_{B} T}{h^{2}}\right)^{3 / 2} e^{\left(E_{r}-E_{F}\right) / k_{B} T} \text { and }
\end{aligned}
$$

where $m_{e}$ and $m_{h}$ are the effective masses of an electron and hole respectively. $E_{V}$ and $E_{C}$ are the energies of the valence and conduction band edges respectively. $E_{F}$ is the energy at the Fermi level.
(i) In some intrinsic semiconductor the effective mass of the electron is $0.07 m_{o}$ and that of the hole is $0.4 m_{o}$ where $m_{0}$ is the rest mass of an electron. Calculate the intrinsic concentration of charge carriers at 300 K . Given: $E_{g}=0.7 \mathrm{eV}$.
(ii) Show that the Fermi level of an intrinsic semiconductor is given by:

$$
E_{F}=\frac{E_{C}+E_{V}}{2}+\frac{3}{4} k_{B} T \ln \frac{m_{h}}{m_{e}}
$$

(iii) What two conditions in (ii) would result in the value of the Fermi level being in the middle of the band gap?
(b) The Hall coefficient for two types of carriers is given by:

$$
R_{H}=\frac{p \mu_{h}^{2}-n \mu_{e}^{2}}{e\left(p \mu_{h}+n \mu_{e}\right)^{2}},
$$

where $p$ and $n$ are the concentrations of holes and electrons respectively, $\mu_{k}$ and $\mu_{e}$ are the respective mobilities.
(i) A sample of germanium shows no Hall effect and the mobility of electrons is 2.1 times that of holes. What is the ratio of the number density of conduction electrons to that of holes for germanium?
(ii) What fraction of the current is carried by holes?
4. (a) Describe the phonon absorption processes in direct and indirect gap semiconductors and explain how the band gap can be determined through these processes.
(b) Using diagrams, explain the steps for the fabrication of an n-p-n transistor by double-diffusion of boron and phosphorous in an $n$-type Si substrate.
(c) The emitter injection efficiency of a p-n-p transistor can be written in terms of the emitter and base properties:

$$
\gamma=\left[1+\frac{L_{p}^{n} n_{n} \mu_{n}^{p}}{L_{n}^{p} p_{p} \mu_{p}^{n}} \tanh \frac{W_{b}}{L_{p}^{n}}\right]^{-1},
$$

where the symbols have their usual meaning and the superscripts indicate which side of the emitter-base junction is referred to, i.e. $L_{p}^{n}$ is the hole diffusion length in the $n$-type base region and $\mu_{n}^{p}$ is the electron mobility in the $p$-type emitter region. The base transport factor is given by:

$$
B=\frac{I_{C}}{I_{E p}} \operatorname{sech} \frac{W_{b}}{L_{p}} .
$$

Assume that this transistor is doped such that the emitter doping is ten times that in the base, the minority carrier mobility in the emitter is one-half that in the base, and the base width is one-tenth the minority carrier diffusion length. The carrier lifetimes are equal. Find the current transfer ratios $\alpha$ and $\beta$; comment on your results.
5. (a) There are several ways of categorizing integrated circuits. Give two categories according to application and two according to fabrication. Briefly describe each category.
(b) One of the most important elements of an integrated circuit is the capacitor. Describe the use of capacitors in dynamic RAM or DRAM cells.
(c) A $3 \mu \mathrm{~m} n$-type epitaxial layer $\left(N_{d}=10^{16}\right)$ is grown on a $p$-type Si substrate. Areas of the $n$ layer are to be junction isolated by a boron diffusion at $1200^{\circ} \mathrm{C}(D=$ $2.5 \times 10^{-12} \mathrm{~cm}^{2} / \mathrm{s}$ ). The surface boron is held constant at $10^{20} \mathrm{~cm}^{-3}$. Given that, erfc $2.75=10^{-4}$ :
(i) What time is required for this isolation diffusion?
(ii) How far does an Sb -doped buried layer ( $D=2 \times 10^{-13} \mathrm{~cm}^{2} / \mathrm{s}$ ) diffuse into the epitaxial layer during this time, assuming the concentration at the substrateepitaxial boundary is constant at $10^{20} \mathrm{~cm}^{-3}$ ?
(d) A Si solar cell has $A=4 \mathrm{~cm}^{2}, I_{t h}=32 \mathrm{nA}$ and $W=1 \mu \mathrm{~m}$. If $g_{o p}=10^{18} \mathrm{EHP} / \mathrm{cm}^{3}$ within $L_{p}=L_{n}=2 \mu \mathrm{~m}$ of the junction, find $I_{s c}$ and $V_{o c}$.
[7]

## APPENDIX A - PHYSICAL CONSTANTS

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Electron rest mass \(m_{e}=9.109 \times 10^{-31} \mathrm{~kg}\)
Proton rest mass \(m_{p}=1.673 \times 10^{-27} \mathrm{~kg}\)
Neutron rest mass \(m_{n}=1.675 \times 10^{-27} \mathrm{~kg}\)
Planck's constant \(h=6.626 \times 10^{-34} \mathrm{Js}^{-1}\)
Planck's constant (reduced) \(\hbar=1.0546 \times 10^{-34} \mathrm{Js}^{-1}\)
Boltzmann constant \(k_{B}=1.381 \times 10^{-23} \mathrm{JK}^{-1}\)
Avogadro's number \(N_{A}=6.022 \times 10^{23}\) per g mole
Bohr magneton \(\mu_{B}=9.274 \times 10^{-24} \mathrm{Am}^{2}\)
Permeability of free space \(\mu_{0}=4 \pi \times 10^{-7} \mathrm{Hm}^{-1}\)
Permittivity of free space \(\varepsilon_{0}=8.85 \times 10^{-12} \mathrm{Fm}^{-1}\)
Electronic charge \(e=1.6 \times 10^{-19} \mathrm{C}\)
Velocity of light \(c=3.00 \times 10^{8} \mathrm{~ms}^{-1}\)
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