UNIVERSITY OF SWAZILAND

MAIN EXAMINATION, SECOND SEMESTER MAY 2012

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER:	INTRODUCTION TO DIGITAL SIGNAL PROCESSING
COURSE CODE:	EE443
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. **Answer any <u>FOUR</u> (4) of the following five questions.**
2. **Each question carries 25 marks.**
3. **Tables of selected window functions and selected Z-transform pairs are attached at the end.**

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THIS PAPER CONTAINS EIGHT (8) PAGES INCLUDING THIS PAGE

QUESTION ONE ***(25 marks)***

(a) A signal $x(t) = 5\sin\omega t$ is digitized using 10 bits.

(i) Calculate the magnitude of the amplitude quantization level. *(3 marks)*

(ii) Calculate the signal-to-quantization-noise ratio (SQNR). *(2 marks)*

(b) For CD quality music assume that $SQNR = 6.02B - 1.6$ dB. It is required that the SQNR be at least 96 dB. What should be the minimum number of bits used in the digitization? *(3 marks)*

(c) A sinusoidal signal of frequency 2 kHz is passed through an 8^{th} order Butterworth filter of cut-off frequency 5 kHz. It is then digitized using 10-bits. Find the minimum sampling frequency if the aliased signal amplitude at 2 kHz should not exceed the rms value of the quantization noise. *(10 marks)*

(d) A digital communications link carries binary coded words representing samples of an input signal $x(t) = 3\cos 600\pi t + 2\cos 1800\pi t$ volts.

(i) Find the Nyquist sampling rate employed. *(2 marks)*

(ii) Find an expression for the discrete-time signal if the signal is sampled at 5 kHz. *(5 marks)*

QUESTION TWO **(25 *marks)***

(a) A linear system with impulse response $h[n]=[1,-2,-3,4]$ has an input $x[n]=[1,1,0,1,1]$. Use the convolution to find its output sequence. *(5 marks)*

(b) A system is described by the difference equation

$$y(n)=\frac{5}{6}y(n-1)-\frac{1}{6}y(n-2)+x(n)-\frac{1}{2}x(n-1)$$

Find a closed form expression for impulse response of the system. Hint: use the z-transform. *(10 marks)*

(c) A system is described by $y(n)=\frac{1}{4}y(n-1)+\frac{1}{8}y(n-2)+x(n)+3x(n-1)+2x(n-2)$.

(i) Express its z-transform as a sum of partial fractions. *(7marks)*

(ii) Systems in the form of a sum of partial fractions can be realized as parallel subsystems where each partial fraction is regarded as a subsystem. Draw the realization of this system in parallel form. *(3 marks)*

QUESTION THREE (25 *marks*)

(a) An IIR filter is described by $y(n) = 0.7y(n-1) + x(n) - 2x(n-1)$. If the sampling frequency is 800 Hz, find

(i) the magnitude, *(9 marks)*

(ii) the phase, *(2 marks)*

of its frequency response at a frequency of 1200 rad/s.

(b) Obtain expressions for the magnitude and phase response of the FIR filter whose impulse response is given by $h[n] = [0.25,\ 0.4,\ 0.8,\ 0.4,\ 0.25]$. *(8 marks)*

(c) An FIR filter has a transfer function $H(z) = 1 + 0.765z^{-1} + z^{-2} = \dfrac{z^2 + 0.765z + 1}{z^2}$.

(i) Sketch the pole-zero diagram of the filter. *(2 marks)*

(ii) Given that the sampling rate is 8 kHz, determine the input frequency that will be maximally attenuated. *(4 marks)*

QUESTION FOUR **(25 *marks*)**

(a) A signal is to be digitally filtered using an IIR filter based on an analogue second-order Butterworth filter with a normalized transfer function $H(s) = \frac{1}{s^2 + \sqrt{2}\,s + 1}$.

Using the Bilinear Transformation, obtain the transfer function of the equivalent low pass digital filter, given a sampling rate of 120 Hz and a cut-off frequency of 20 Hz. Your calculations should be accurate to 4 decimal places. *(15 marks)*

(b) Obtain the DFT of the sequence x(n) = [1, -1, 2, 2]. *(10 marks)*

QUESTION FIVE **(25 *marks)***

(a) Using the windowed sinc method with a Hamming window, determine the coefficients of a 7-tap linear-phase FIR low pass filter. The filter is to have a cut-off frequency of 3 kHz and sampling frequency of 16 kHz. *(15 marks)*

(b) A discrete-time system has the following pole-zero decription:

Zeros: $z = +j1.5$ and $z = -j1.5$

Poles: $z = 0.8$, $z = 0.5 + j0.5$ and $z = 0.5 - j0.5$

(i) Is the system stable? Give a reason for your answer. *(2 marks)*

(ii) Derive the difference equation of the system. *(8 marks)*

TABLE OF Z-TRANSFORMS OF SOME COMMON SEQUENCES

Discrete-time sequence $x(n),\ n \geq 0$	**Z-transform** $H(z)$
$k\delta(n)$	k
k	$\frac{kz}{z-1}$
$ke^{-\alpha n}$	$\frac{kz}{z-e^{-\alpha}}$
$k\alpha^n$	$\frac{kz}{z-\alpha}$
kn	$\frac{kz}{(z-1)^2}$
kn^2	$\frac{kz(z+1)}{(z-1)^3}$
$kn\alpha^n$	$\frac{k\alpha z}{(z-\alpha)^2}$

QUANTIZATION

For a sine wave $SQNR = 6.02B + 1.76$ dB.

LOW PASS TO LOW PASS TRANSFORMATION

$s = \frac{s}{\omega_p'}$ where pre-warped frequency $\omega_p' = \tan\left(\frac{\pi f_c}{f_s}\right)$

SUMMARY OF IMPORTANT FEATURES OF SELECTED WINDOW FUNCTIONS

Name of Widow	Normalized Transition Width	Passband Ripple (dB)	Main lobe relative to Sidelobe (dB)	Max. Stopband attenuation (dB)	6 dB normalized bandwidth (bins)	Window Function $\omega(n)$, $\lvert n \rvert \le (N-1)/2$
Rectangular	0.9/N	0.7416	13	21	1.21	1
Hanning	3.1/N	0.0546	31	44	2.00	$0.5+0.5\cos\left(\frac{2\pi n}{N}\right)$
Hamming	3.3/N	0.0194	41	53	1.81	$0.54+0.46\cos\left(\frac{2\pi n}{N}\right)$
Blackman	5.5/N	0.0017	57	74	2.35	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$
Kaiser	2.93/N (β=4.54)	0.0274		50		$\frac{I_o\left(\beta\left\{1-\left[\frac{2n}{N-1}\right]^2\right\}^{\frac{1}{2}}\right)}{I_o(\beta)}$
	4.32/N (β=6.76	0.00275		70		
	5.71/N (β=8.96)	0.000275		90		

$$\text{Bin width} = \frac{f_s}{N}\ \text{Hz}$$