UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, JULY 2012

FACULTY OF SCIENCE

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER:	INTRODUCTION TO DIGITAL SIGNAL PROCESSING
COURSE CODE:	EE443
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. **Answer any <u>FOUR</u> (4) of the following five questions.**
2. **Each question carries 25 marks.**
3. **Tables of selected window functions and selected Z-transform pairs are attached at the end.**

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THIS PAPER CONTAINS EIGHT (8) PAGES INCLUDING THIS PAGE

QUESTION ONE *(25 marks)*

(a) A signal $x(t) = 6\sin\omega t$ volts is digitized using 12 bits.

(i) Calculate the magnitude of the amplitude quantization interval. *(3 marks)*

(ii) Calculate the signal-to-quantization-noise ratio (SQNR). *(2 marks)*

(b) A system is described by the difference equation

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 2x(n-1)$$

Use the Z-Transform and its inverse to find the impulse response of the system. *(10 marks)*

(c) A system has an impulse response $h(n) = [0, -1, 2, 1, 0]$. A discrete signal $x(n) = [3, 2, -1]$ is passed through the system. Find the output sequence. *(5 marks)*

(d) A system has a transfer function $H(z) = z^{-1} + 2z^{-2}$. A sequence whose Z-Transform is $X(z) = 1 + 2z^{-1} + z^{-2}$ is passed through the system. Find the output sequence. *(5 marks)*

QUESTION TWO (25 *marks)*

(a) For the system described by $y(n) = x(n) + 3x(n-2) + 0.6y(n-1)$

(i) Find the poles and zeros. *(4 marks)*

(ii) Determine whether the system is stable or not. *(1 mark)*

(b) For the system with impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n-1) + 3^n u(n)$

(i) Find the poles and zeros. *(7 marks)*

(ii) Determine whether the system is stable or not. *(1 mark)*

(c) A discrete-time system has a transfer function $H(z) = \frac{z}{z+0.8}$.

(i) What is the Region of Convergence of this function? *(2 marks)*

(ii) Obtain expressions for the magnitude and phase of its frequency response. *(6 marks)*

(iii) Evaluate its magnitude and phase at $\frac{1}{3}$ of its sampling frequency. *(4 marks)*

QUESTION THREE **(25 *marks)***

(a) Find the DFT of the sequence [2, 1, 2, -1]. *(10 marks)*

(b) A 1st order analogue low pass filter has a normalized transfer function $H(s)=\frac{1}{s+1}$.

(i) Given a sampling rate of 12 kHz and a cut off frequency of 2.5 kHz, use the Bilinear transform, obtain the transfer function of the equivalent low pass digital filter. *(13 marks)*

(ii) Draw a realization structure of the equivalent digital filter. *(2 marks)*

QUESTION FOUR **(25 *marks*)**

(a) (i) Find simplified expressions for the magnitude and phase response of the FIR filter whose impulse response is given by $h[n]=[-0.4,\ 0.6,\ 0.8,\ 0.6,\ -0.4]$. *(8 marks)*

(ii) How would you describe this FIR filter? *(1 mark)*

(b) An IIR filter has a transfer function $H(z)=\dfrac{z^2+1}{z^2+0.64}$.

(i) Sketch the pole-zero diagram of the filter. *(2 marks)*

(ii) Given that the sampling rate is 16 kHz, determine the input frequency that will be maximally attenuated. *(4 marks)*

(c) A discrete system has a transfer function $H(z)=\dfrac{k(z+1)}{(z-0.6)}$.

(i) Write down an expression for its frequency response. *(2 marks)*

(ii) Obtain a simplified expression for its frequency response at $\frac{1}{4}$ of the sampling frequency. *(4 marks)*

(iii) Determine the value of k if the magnitude of the transfer function at $\frac{1}{4}$ of the sampling frequency is 2. *(4 marks)*

QUESTION FIVE **(25 *marks*)**

(a) Using the windowed sinc method with a Hanning window, determine the coefficients of a 9-tap ($N = 9$) linear-phase FIR low pass filter. The filter is to have a cut-off frequency of 5 kHz and sampling frequency of 16 kHz. *(20 marks)*

(b) A discrete-time system has the following frequency response

$$H(e^{j\Omega}) = \frac{1 - \frac{1}{2}e^{-j\Omega} + e^{-j3\Omega}}{1 + \frac{1}{2}e^{-j\Omega} + \frac{3}{4}e^{-j2\Omega}}$$

Derive the difference equation of the system. *(5 marks)*

TABLE OF Z-TRANSFORMS OF SOME COMMON SEQUENCES

Discrete-time sequence $x(n),\ n \geq 0$	**Z-transform** $H(z)$
$k\delta(n)$	k
k	$\frac{kz}{z-1}$
$ke^{-\alpha n}$	$\frac{kz}{z-e^{-\alpha}}$
$k\alpha^n$	$\frac{kz}{z-\alpha}$
kn	$\frac{kz}{(z-1)^2}$
kn^2	$\frac{kz(z+1)}{(z-1)^3}$
$kn\alpha^n$	$\frac{k\alpha z}{(z-\alpha)^2}$

QUANTIZATION

For a sine wave $SQNR = 6.02B + 1.76\,\text{dB}$.

LOW PASS TO LOW PASS TRANSFORMATION

$s = \frac{s}{\omega_p'}$ where pre-warped frequency $\omega_p' = \tan\left(\frac{\pi f_c}{f_s}\right)$

SUMMARY OF IMPORTANT FEATURES OF SELECTED WINDOW FUNCTIONS

Name of Widow	Normalized Transition Width	Passband Ripple (dB)	Main lobe relative to Sidelobe (dB)	Max. Stopband attenuation (dB)	6 dB normalized bandwidth (bins)	Window Function $\omega(n)$, $\lvert n \rvert \le (N-1)/2$
Rectangular	0.9/N	0.7416	13	21	1.21	1
Hanning	3.1/N	0.0546	31	44	2.00	$0.5+0.5\cos\left(\frac{2\pi n}{N}\right)$
Hamming	3.3/N	0.0194	41	53	1.81	$0.54+0.46\cos\left(\frac{2\pi n}{N}\right)$
Blackman	5.5/N	0.0017	57	74	2.35	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$
Kaiser	2.93/N (β=4.54)	0.0274		50		$\frac{I_o\left(\beta\left\{1-\left[\frac{2n}{N-1}\right]^2\right\}^{\frac{1}{2}}\right)}{I_o(\beta)}$
	4.32/N (β=6.76	0.00275		70		
	5.71/N (β=8.96)	0.000275		90		

$$\text{Bin width} = \frac{f_s}{N}\ \text{Hz}$$