# UNIVERSITY OF SWAZILAND MAIN EXAMINATION, DECEMBER 2012 

FACULTY OF SCIENCE AND ENGINEERING DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER:<br>BASIC ELECTRICAL ENGINEERING<br>COURSE NUMBER:<br>EE251<br>TIME ALLOWED:<br>THREE HOURS

INSTRUCTIONS:

1. There are five questions in this paper. Answer any FOUR questions.
2. Each question carries 25 marks.
3. Marks for different sections are shown on the right hand margin.
4. Show the steps clearly in all your calculations including any assumptions made.
5. A sheet containing useful formulae is attached at the end of the question paper.

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

## QUESTION 1 ( 25 marks)

(a) A planar resistor of uniform thickness is made by cutting up squares of material from a slab of resistance material. Show that the resistance between any two opposite ends of the planar resistor is the same regardless of the size of the square cut.
(b) Determine the resistance of 1500 m of copper cable having a diameter of 12 mm , given that the resistivity of copper is $1.7 \times 10^{-8} \Omega-\mathrm{m}$.
(c) A Li-Ion battery for a digital camera is rated at $3.6 \mathrm{~V}, 2.4 \mathrm{~W}$-h. When viewing a video taken by the camera 250 mA is taken from the battery. Estimate how long a fully charged battery can last if used for viewing videos.
(d) The Swaziland Electricity Company supplies residential premises with $230 \mathrm{~V}, 50$ Hz mains and charges E 0.84 for every kW -h of energy consumed. A student has used electricity on a typical day in his premises as follows:

Made 4 slices of toast using a 900 W toaster which takes 2 min to toast each pair of slices

Boiled 4.5 litres of water using $1800 \mathrm{~W}, 1.5$ litre capacity kettle which takes 5 $\min$ to boil a full kettle of water.

Used a $230 \mathrm{~V}, 60 \mathrm{~W}$ soldering iron for 24 min .
Used two 14 W energy saving bulbs for 6 h
Charged a cell phone for $1 \frac{1}{2} \mathrm{~h}$ using a $230 \mathrm{~V}, 200 \mathrm{~mA}$ charger.
Used a 1220 W steam iron for 24 min
Although the music system was not used, it was left on standby for 24 h . The system draws 20 mA from the 230 V mains while on standby.

Find the following:
(i) The total cost of electricity used.
(ii) The input current to the premises if all these appliances are switched on simultaneously and the overall power factor is 0.9 lagging. (3 marks)

## QUESTION 2 (25 marks)

(a) Use Delta-Star or Star-Delta transformation to evaluate the current supplied by the battery in Fig.Q.2a.
(10 marks)


Fig. Q.2a
(b) Using the mesh (loop) current analysis method find the voltage across the current source shown in Fig.Q.2b. Note that the circuit has a voltage dependent voltage source, $3 V_{x}$.

(15 marks)

Fig. Q.2b

## QUESTION 3 (25 marks)

(a) Consider the circuit shown in Fig. Q.3a.
(i) Find the Thevenin equivalent of the circuit across the points $\mathbf{a}$ and $\mathbf{b}$.
(8 marks)
(ii) If a variable resistor were connected between the points $\mathbf{a}$ and $\mathbf{b}$, what will be the maximum power dissipated in the variable resistor?


Fig. Q.3a
(b) Use nodal analysis to find the node voltages $V_{\mathrm{a}}, V_{\mathrm{b}}$ and $V_{\mathrm{c}}$ shown in Fig.Q.3b. (15 marks)


Fig. Q.3b

## QUESTION 4 (25 marks)

(a) A $500-\mu \mathrm{F}$ capacitor connected in series with a $250-\Omega$ resistor. The circuit is connected across a 50 V dc supply via a switch which is switched on at $t=0 \mathrm{~s}$. Determine:
(i) the initial value of the current flowing.
(ii) the value of current in the circuit 100 ms after switching on.
(iii) the voltage across the capacitor, 50 ms after switching on.
(iv) the time at which the voltage across the resistor is 20 V .
(b) A power supply has a voltage $v_{s}=308 \sin (314 t)$ volts. This supply supplies currents $i_{1}$ and $i_{2}$ to two parallel branches with complex impedances $Z_{1}$ and $Z_{2}$ as shown in Fig. Q4b where:

$$
\begin{aligned}
& i_{1}=14.14 \sin \left(314 t+45^{\circ}\right) \mathrm{A} \\
& i_{2}=21.21 \sin \left(314 t-60^{\circ}\right) \mathrm{A}
\end{aligned}
$$

(i) Using phasors, or otherwise, derive a sinusoidal expression for the total current taken from the supply.
(ii) Determine the complex impedance connected to each branch.


Fig. Q.4b

QUESTION 5 (25 marks)
(a) A circuit consisting of a coil of inductance 0.15 H and resistance $12 \Omega$, in series with a capacitor of $12 \mu \mathrm{~F}$ is connected to a variable frequency supply which has a constant voltage 24 V. Determine:
(i) the resonance frequency of the circuit. (2 marks)
(ii) the current in the circuit at resonance. (2 marks)
(iii) the voltages across the inductance and the capacitor at resonance. (3 marks)
(iv) the Q -factor of the circuit.
(b) A coil of inductance 80 mH and resistance $120 \Omega$ is connected to a $230 \mathrm{~V}, 50 \mathrm{~Hz}$ supply. In parallel with it is a $60-\mu \mathrm{F}$ capacitor in series with a $40 \Omega$ resistor as shown in Fig. Q5b. Determine:
(i) The total impedance of the circuit,
(ii) The power factor of the combined circuit stating whether it is leading or lagging.
(iii) The total Active Power taken from the supply,
(iv) The total Reactive Power supplied,
(v) The Apparent Power supplied.


Fig. Q5b

## SOME SELECTED FORMULAE

$$
\begin{aligned}
& R=\rho \frac{l}{a} \\
& P=V_{r m s} I_{r m s} \cos \phi, \quad V_{r m s}=\frac{V_{m}}{\sqrt{2}}, \quad P^{2}=Q^{2}+R^{2} \\
& Z=\frac{\vec{V}}{\vec{I}}=R+j X \\
& R=\frac{V_{R}}{I_{R}}, \quad X_{L}=\left|\frac{V_{L}}{I_{L}}\right|=\omega L, \quad X_{C}=\left|\frac{V_{C}}{I_{C}}\right|=\frac{1}{\omega C} \\
& W_{L}=\frac{1}{2} L I^{2} \quad W_{C}=\frac{1}{2} C V^{2} \\
& \omega_{o}^{2}=L C, \quad Q=\omega_{o} L / R \\
& R_{1}=\frac{R_{B} R_{C}}{R_{A}+R_{B}+R_{C}}, \quad R_{A}=\frac{R_{1} R_{2}+R_{2} R_{3}+R_{1} R_{3}}{R_{1}} \\
& v_{C}(t)=V_{s}\left(1-e^{-t / R C}\right), \quad v_{C}(t)=V_{i} e^{-t / R C}
\end{aligned}
$$

