University of Swaziland
Faculty of Science and Engineering
Department of Electrical and Electronic Engineering

Main Examination 2012

Title of Paper:	Signals and Systems I
Course Number:	EE331
Time Allowed:	3 hrs

Instructions:

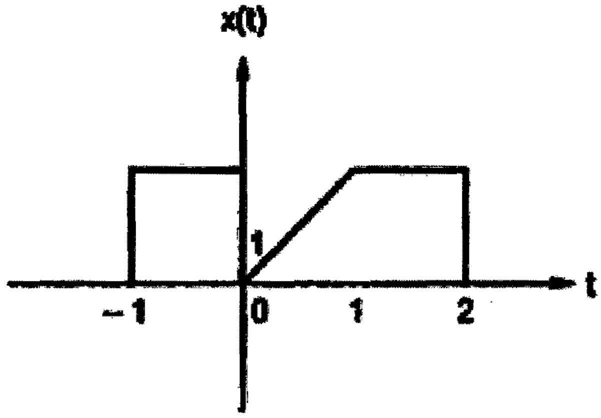
1. Answer any four (4) questions.
2. Each question carries 25 marks.
3. Useful tables are attached at the end of the question paper

This paper should not be opened until permission has been given by the invigilator.

This paper contains nine (9) pages including this page.

Question 1

a) From the given signal $x(t)$ as shown below, plot $x\left(1-\frac{t}{2}\right)$ [3]



b) Compute the impulse response, $h(t)$ of a system with the transfer function, $H(s)$, as given below. Assume the input $x(t)$ and output $y(t)$ are related by $Y(s)=H(s)X(s)$. [10]

$$H(s)=\frac{(s+4)}{s\left(s^2+5s+6\right)}$$

c) Find the fundamental period (in seconds) of the sum signal: [3]

$$x(t)=3\sin\left(\frac{\pi t}{3}\right)+5\cos\left(\frac{\pi t}{5}\right)+7\sin\left(\frac{\pi t}{7}\right)$$

d) For $x[n]=\delta[n]+2\delta[n-1]-\delta[n-3]$ and $h[n]=2\delta[n+1]+2\delta[n-1]$, compute $y[n]=x[n]*h[n]$. [3]

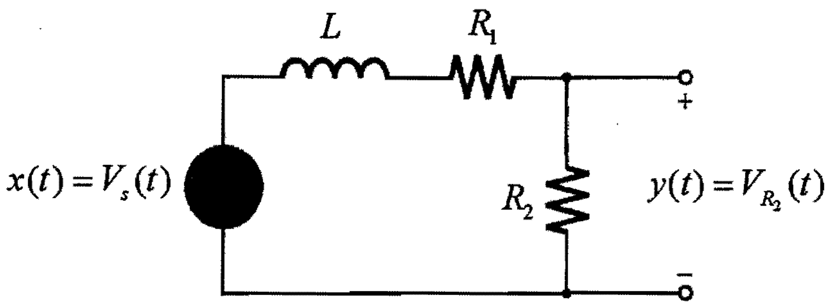
e) Using the graphical method, compute, $y(t)=x(t)*h(t)$ for $x(t)=u(t-3)-u(t-5)$ and $h(t)=e^{-3t}u(t)$, [6]

Question2

a) Determine whether the following DT (discrete-time) signal is periodic or not? If periodic, determine fundamental period: [3]

$$\cos\left[\frac{2\pi n}{5}\right]+\cos\left[\frac{2\pi n}{7}\right]$$

b) Derive the differential equation in terms of $x(t)$ and $y(t)$ that relates all the circuit components in the circuit below: [10]



c) State if the following systems are linear/non-linear, causal/non-causal, time-invariant/ time-varying:

i) $y[n]=n^2x[n+2]$ [3]

ii) $y(t)=A\cos\left(2\pi ft+x(t)\right)$ [3]

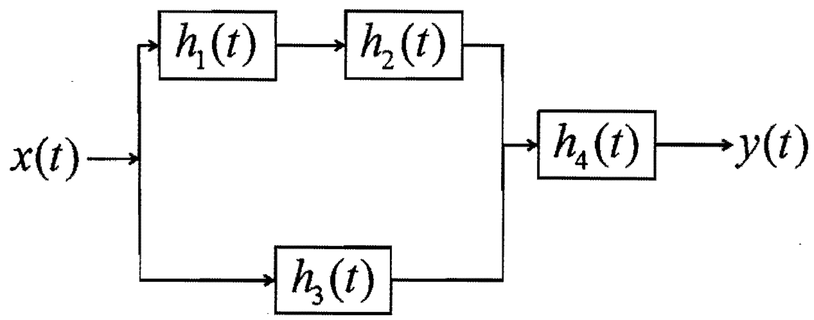
d) State the sifting property of the unit impulse, $\delta(t)$ [1]

e) Solve $\int_{-\infty}^{\infty} x(t)\delta(t-4)dt$ [1]

f) For the following signal find an expression for its second derivative: [2]

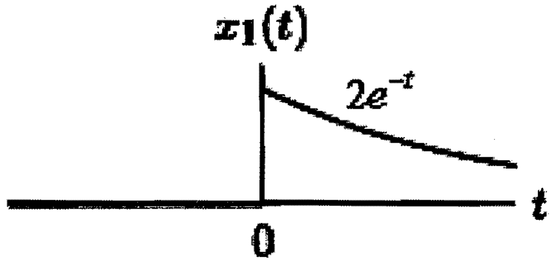
$$x(t)=2r(t)-3r(t-1)-4r(t-3)$$

g) Determine the overall impulse response of the following LTI system: [2]



Question 3

a) Find and sketch the region of convergence (ROC) of the following signal: [4]



b) Describe briefly in words each of the following signal operations:

i) $x\left(\frac{t}{3}+1\right)$ [2]

ii) $x(-t+2)$ [2]

c) Determine the system response of the following system:

$$5\frac{dy(t)}{dt}+10y(t)=2x(t)$$

for the input $x(t)=2u(t)$, assume zero initial conditions. [10]

d) Draw a block diagram representation for the causal LTI system described by the following difference equation:

$$y[n]=\frac{1}{3}y[n-1]+x[n-1]$$

[3]

e) Differentiate between an invertible and a noninvertible system, give suitable examples for each. Do not consider any of the systems defined in this exam as your examples.

[4]

Question 4

a) Given the following differential equation, find $y(t)$: [5]

$$\frac{d^2y(t)}{dt}+3\frac{dy(t)}{dt}+2y(t)=\delta(t)$$

b) The following signal:

$$x(t)=e^{-t}, \quad -1\le t\le 1$$

has these three basis functions,

$$\phi_0(t)=1, \quad \phi_1(t)=t, \quad \phi_2(t)=\tfrac{3}{2}t^2-\tfrac{1}{2}, \quad -1\le t\le 1$$

and the corresponding energies are:

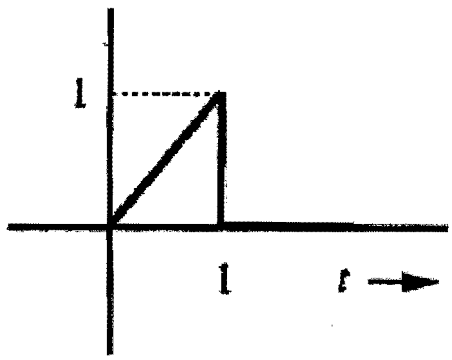
$$E_0=2, \quad \mathrm{E}_1=\frac{2}{3}, \quad \mathrm{E}_2(t)=\frac{2}{7}.$$

Find the approximate value of $x(t)$ using the minimum square error method. [10]

c) Find the Laplace transforms of the following signals:

i) $f(t)=\sin\left[\omega_0(t-\tau)\right]u(t-\tau)$ [2]

ii)



[3]

d)

i) Write a mathematical expression for the Signum function, $\mathrm{sgn}(t)$. [3]

ii) Write the Signum function in terms of the step function. [2]

Question 5

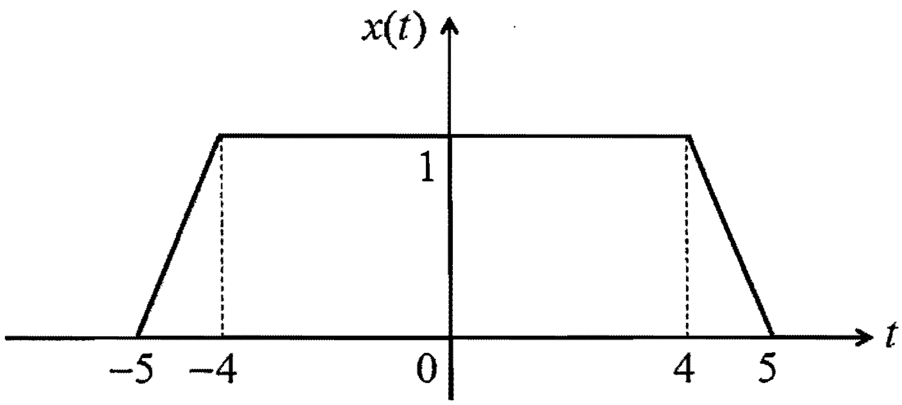
a) Find the Fourier series coefficients of the signal: [5]

$$x(t) = \sin^3(3\pi t)$$

b) Determine whether the following signals are energy signals or power signals and calculate their energy or power.

i) $x[n] = \left(\frac{1}{2}\right)^n u[n]$ [3]

ii)

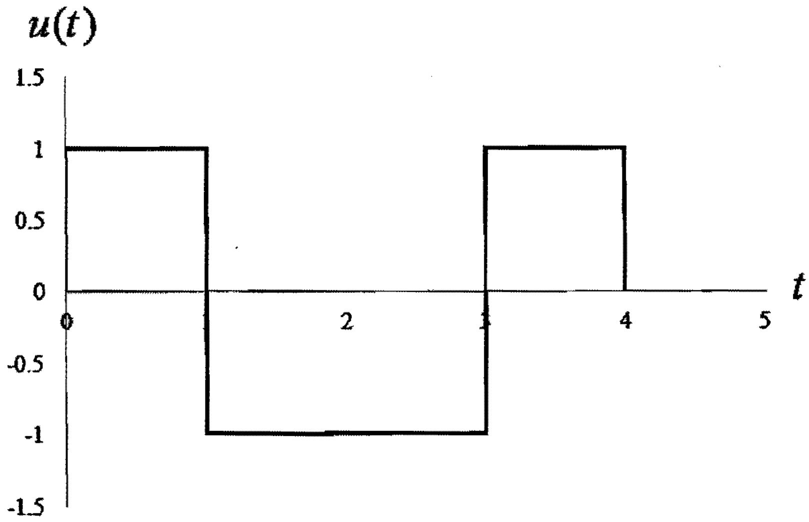


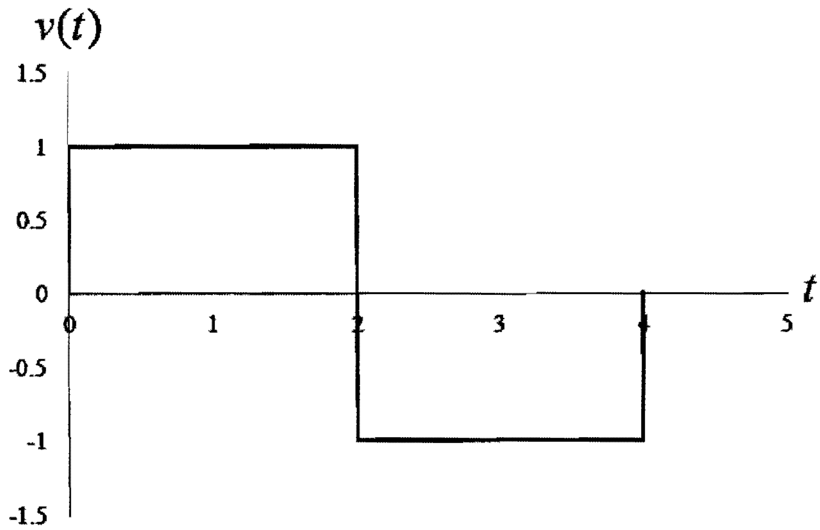
[5]

c) Find the even and odd components of the following: [4]

$$x(t) = \cos^2\left(\frac{\pi t}{2}\right)$$

d) Is the following pair of signals orthogonal over the interval (0,4)? Prove your answer. [4]





e) State Parseval's theorem. [2]

f) For the time function:

$$f(t) = \int_0^t \tau \sin\left(2\left(t-\tau\right)\right) d\tau, \quad t \geq 0$$

Use the Initial Value Theorem to find $f(0)$. [2]

Table of Laplace Transforms

delta function	$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1
shifted delta function	$\delta(t-a)$	$\xleftrightarrow{\mathcal{L}}$	e^{-as}
unit step	$u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$
ramp	$tu(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2}{s^3}$
n-th power	t^n	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\lvert t\rvert}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\xleftrightarrow{\mathcal{L}}$	$-F'(s)$
frequency n-th differentiation	$t^nf(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^nF^{(n)}(s)$
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

i) Time-shift (delay): $f(t-t_0) \xleftrightarrow{L} F(s)e^{-st_0}, \quad t_0 > 0$

ii) Time differentiation: $\dfrac{df(t)}{dt} \xleftrightarrow{L} sF(s) - f(0)$

iii) Time integration: $\int_0^t f(t)dt \xleftrightarrow{L} \dfrac{F(s)}{s}$

iv) Linearity: $af(t) + bg(t) \xleftrightarrow{L} aF(s) + bF(s)$

v) Convolution Integral: $x(t) * h(t) \xleftrightarrow{L} X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t} f(t) \xleftrightarrow{L} F(s-\alpha)$

vii) Multiplying by t: $tf(t) \xleftrightarrow{L} -\dfrac{dF(s)}{ds}$

viii) Scaling: $f(at) \xleftrightarrow{L} \dfrac{1}{a} F\left(\dfrac{s}{a}\right), \quad a > 0$

ix) Initial Value Theorem: $\lim_{s\to\infty}\{sF(s)\} = f(0)$

x) Final Value Theorem: $\lim_{s\to 0}\{sF(s)\} = \lim_{t\to\infty} f(t)$

Standard Table of Forced Response or Particular Solutions

	Input	**Particular Solution**
1	$cx^m(t)$	$a_0 + a_1x(t) + \ldots + a_mx^m(t)$
2	$cx^m(t)e^{\alpha x(t)}$	$\left(a_0 + a_1x(t) + \ldots + a_mx^m(t)\right)e^{\alpha x(t)}$
3	$cx^m(t)\cos\left(bx(t)\right)$	$\left(a_0 + a_1x(t) + \ldots + a_mx^m(t)\right)\cos\left(bx(t)\right) + \left(c_0 + c_1x(t) + \ldots + c_mx^m(t)\right)\sin\left(bx(\right.$
4	$cx^m(t)\sin\left(bx(t)\right)$	$\left(a_0 + a_1x(t) + \ldots + a_mx^m(t)\right)\sin\left(bx(t)\right) + \left(c_0 + c_1x(t) + \ldots + c_mx^m(t)\right)\cos\left(bx(\right.$

where $c, a_0, a_1, a_m, c_0, c_1, c_m$ are constants.