University of Swaziland
Faculty of Science and Engineering
Department of Electrical and Electronic Engineering

Main Examination 2013

Title of Paper:	Signals and Systems II
Course Number:	EE332
Time Allowed:	3 hrs

Instructions:

1. Answer any four (4) questions
2. Each question carries 25 marks
3. Useful tables are attached at the end of the question paper
4. Linear-log paper is attached to the question paper

This paper should not be opened until permission has been given by the invigilator.

This paper contains nine (9) pages including this page.

Question 1

a) Based on the system $G(s)$ shown below.

$$G(s)=\frac{10(s+1)}{s\left(s^2+60s+800\right)}$$

Sketch the magnitude and phase Bode plots. Showing all the elements involved in your construction. [20]

b) Plot the magnitude and phase spectrum of the voltage signal:

$$v(t)=2-3\cos(4t)+5\cos(6t+45°)+\sin(8t-75°).$$

[5]

Question 2

a) For Fig. Q2:

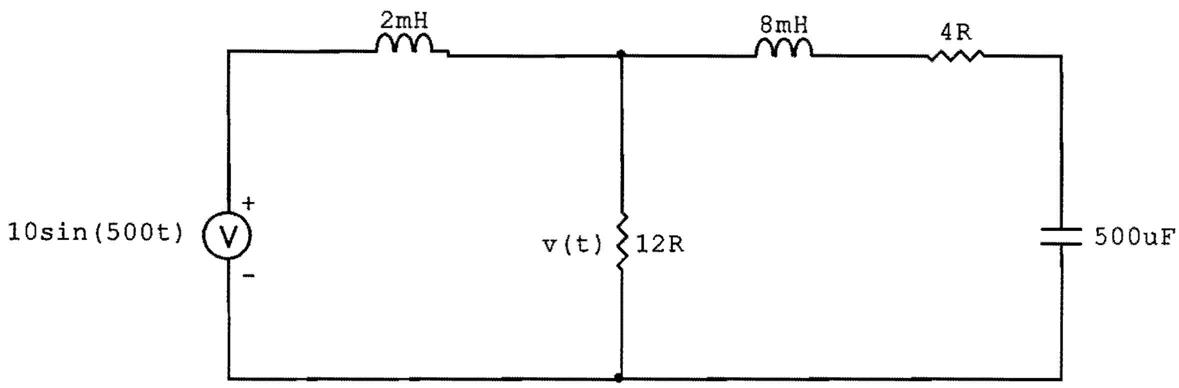
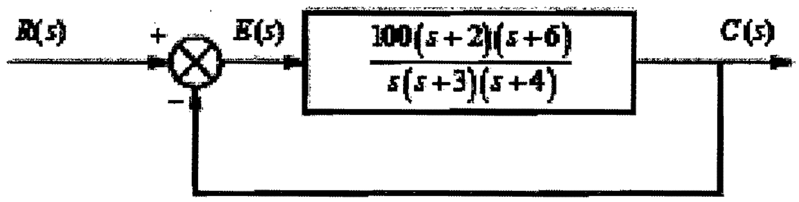


Fig. Q2

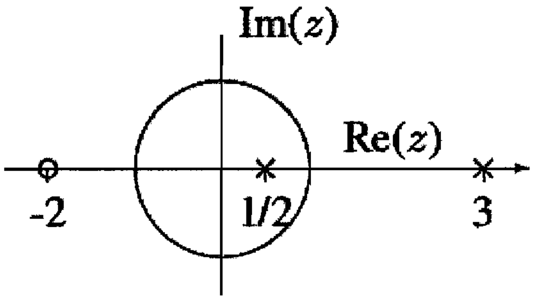
i) Find the equivalent impedance in phasor form. [5]

i) Determine the steady-state voltage, $v(t)$ across the 12Ω resistor, using the phasor form. [5]

b) Find the steady state error for an input of $5t^2$ into the system shown below: [3]

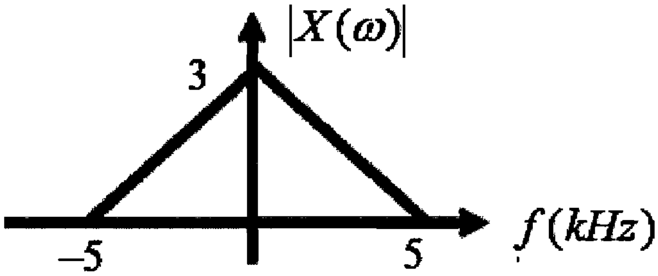


c) A z-transform, $X(z)$ has the following pole-zero plot. Find an expression for $X(z)$. [2]



d) Draw a simple block diagram showing how amplitude modulation occurs. Sketch all the input signals and the overall output signal. [5]

e) Let $x(t)$ be a signal with spectrum shown below:



Sketch the spectrum of the sampled signal, $|X_s(\omega)|$, assuming we sample at $T_s = 25\mu s$. Show all your working. [5]

Question 3

A unit step is applied to a system with the transfer function, $H(s) = \dfrac{14}{s^2 + 5s + 28}$. Find the following:

a) The roots of the characteristic equation and general solution [3]
b) The undamped natural frequency, ω_n [2]
c) The damping ratio, ζ [2]
d) The damped natural frequency, ω_d [3]
e) The stability ratio, σ [3]
f) The percentage overshoot [3]
g) The settling time, t_s [3]
h) The peak time, t_p [3]
i) The DC gain, K [1]
j) Is this system underdamped, overdamped or critically damped? Explain your answer? [2]

Question 4

a) A 1.5 MHz carrier is amplitude modulated by three sinusoidal signals of frequency 500 Hz, 800 Hz and 1,400 Hz. What are the frequencies in the AM spectrum? [3]

b) An AM signal is represented by the equation:

$$v(t) = \left(15 + 3\sin\left(2\pi \times 5 \times 10^3 t\right)\right) \times \sin\left(2\pi \times 0.5 \times 10^6 t\right) volts$$

i) What are the values of the carrier and modulating frequencies? [2]

ii) What are the amplitudes of the carrier and of the upper and lower sideband frequencies? [3]

iii) What is the bandwidth of this signal? [2]

c)

i) What type of filter is shown in Fig. Q4.1? [1]

ii) Find expressions for the impulse and step responses for this filter. [9]

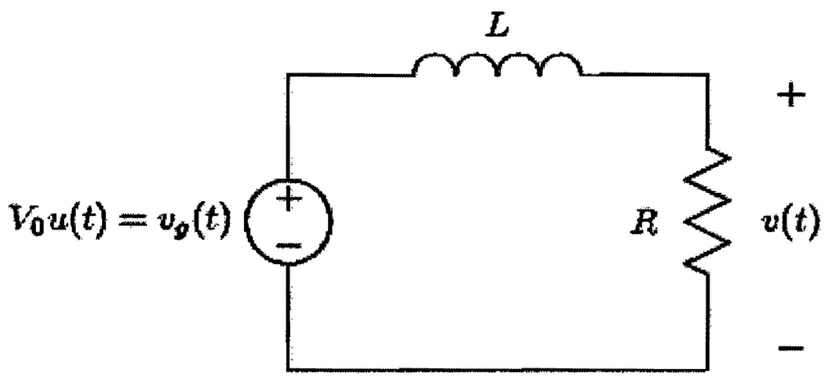


Fig. Q4.1

d) Consider Fig. Q4.2. Does the Nyquist sampling rate hold? Explain your answer. [2]

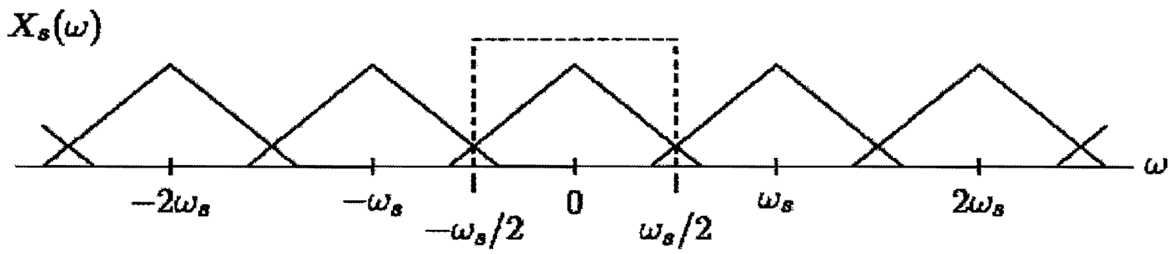


Fig. Q4.2

e) Determine the final value of a signal $x(t)$ having the z-transform:

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-e^{-aT}z^{-1}}, \quad a > 0$$

[3]

Question 5

a) Solve the following difference equation using z-transforms:

$$x[n+2]+3x[n+1]+2x[n]=0, \quad x[0]=0, \quad x[1]=1.$$

[10]

b) With the aid of suitable sketches, explain the impulse reconstruction of a signal from its samples. [4]

c) Obtain the z-transform of:

$$X(s)=\frac{1}{s(s+1)}$$

[3]

d) Define the following terms as used in the analysis of linear signals and systems:

i) Interpolation [2]

ii) Time delay, t_d [2]

iii) Transient and steady state responses [2]

iv) The velocity constant, K_v. [2]

Table of Laplace Transforms

delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\|t\|}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$
frequency n-th differentiation	$t^nf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^nF^{(n)}(s)$
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

Time-shift (delay): $f(t-t_0)\overset{L}{\longleftrightarrow}F(s)e^{-st_0},\quad t_0>0$

Time differentiation: $\dfrac{df(t)}{dt}\overset{L}{\longleftrightarrow}sF(s)-f(0)$

Time integration: $\displaystyle\int_0^t f(t)dt\overset{L}{\longleftrightarrow}\frac{F(s)}{s}$

Linearity: $af(t)+bg(t)\overset{L}{\longleftrightarrow}aF(s)+bF(s)$

Convolution Integral: $x(t)*h(t)\overset{L}{\longleftrightarrow}X(s)H(s)$

Frequency-shift: $e^{\alpha t}f(t)\overset{L}{\longleftrightarrow}F(s-\alpha)$

Multiplying by t: $tf(t)\overset{L}{\longleftrightarrow}-\dfrac{dF(s)}{ds}$

Scaling: $f(at)\overset{L}{\longleftrightarrow}\dfrac{1}{a}F\left(\dfrac{s}{a}\right),\quad a>0$

Initial Value Theorem: $\lim_{s\to\infty}\{sF(s)\}=f(0)$

Final Value Theorem: $\lim_{s\to 0}\{sF(s)\}=\lim_{t\to\infty}f(t)$

Table of Z-Transforms

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n) z^{-n}$	
2	$\delta(n)$	1	$\lvert z \rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z \rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z \rvert > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z \rvert > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\lvert z \rvert > \lvert a \rvert$
7	$e^{-na}u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z \rvert > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z \rvert > \lvert a \rvert$
9	$\sin(an)u(n)$	$\frac{z\sin(a)}{z^2 - 2z\cos(a) + 1}$	$\lvert z \rvert > 1$
10	$\cos(an)u(n)$	$\frac{z[z-\cos(a)]}{z^2 - 2z\cos(a) + 1}$	$\lvert z \rvert > 1$
11	$a^n \sin(bn)u(n)$	$\frac{[a\sin(b)]z}{z^2 - [2a\cos(b)]z + a^2}$	$\lvert z \rvert > \lvert a \rvert$
12	$a^n \cos(bn)u(n)$	$\frac{z[z - a\cos(b)]}{z^2 - [2a\cos(b)]z + a^{-2}}$	$\lvert z \rvert > \lvert a \rvert$
13	$e^{-an}\sin(bn)u(a)$	$\frac{[e^{-a}\sin(b)]z}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z \rvert > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\frac{z[z - e^{-a}\cos(b)]}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z \rvert > e^{-a}$

Properties of Z-Transforms

Linearity: $ax_1[k] + bx_2[k] \Leftrightarrow aX_1(z) + bX_2(z)$

Time Reversal: $x[-k] \Leftrightarrow X(1/z)$

Summation: $\sum_{n=-\infty}^{k} x[n] \Leftrightarrow \frac{zX(z)}{z-1}$

Initial Value: $x[0] = \lim_{z \to \infty} X(z)$

Final Value: $x[\infty] = \lim_{z \to 1} (z-1)X(z)$

Convolution: $x[k] * h[k] \Leftrightarrow X(z)H(z)$

Differencing: $x[k] - x[k-1] \Leftrightarrow \left(1 - z^{-1}\right)X(z)$

Differentiation: $-kx[k] \Leftrightarrow z\frac{d}{dz}X(z)$

Time Shifting: $x[n - n_o] \Leftrightarrow z^{-n_o}X(z), n_o \geq 0$

$$x[n + n_o] \Leftrightarrow z^{n_o}\left(X(z) - \sum_{m=0}^{n_0 - 1} x[m] z^{-m}\right), n_o \geq 0$$