UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION, JULY 2013

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER:	INTRODUCTION TO DIGITAL SIGNAL PROCESSING
COURSE CODE:	EE443
TIME ALLOWED:	THREE HOURS

INSTRUCTIONS:

1. **Answer any <u>FOUR</u> (4) of the following five questions.**
2. **Each question carries 25 marks.**
3. **Tables of selected window functions and selected Z-transform pairs are attached at the end.**

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THIS PAPER CONTAINS EIGHT (8) PAGES INCLUDING THIS PAGE

QUESTION 1 ***(25 marks)***

(a) Figure Q.1a shows the block diagram of a simplified digital signal processing scheme

(i) Explain its purpose each block. *(5 marks)*

(ii) Describe the signal coming out of each block specifying whether it is discrete or continuous (in time or amplitude). *(5 marks)*

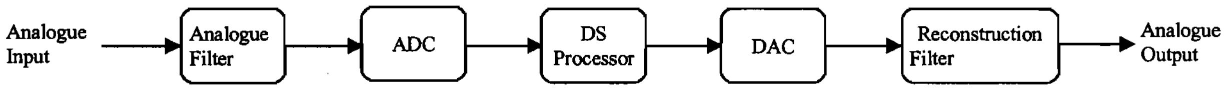

Fig.Q1a

(b) A signal has a flat uniform spectrum. A second order (n=2) Butterworth filter with cut-off frequency of 3.4 kHz is used to filter this signal. The filtered signal is then sampled at 8 kHz. Determine

(i) The percentage of aliased signal to non-aliased signal at the cut-off frequency. *(3 marks)*

(ii) The percentage of aliased to non-aliased signal at 1 kHz. *(3 marks)*

(iii) The r.m.s value of the quantization noise, if a signal $5\sin(6800\pi t)$ volts is passed through the filter and then quantized using 12 bits. *(3 marks)*

An *n*th order Butterworth analogue filter has a magnitude response $\dfrac{1}{\sqrt{1+\left(\dfrac{f}{f_c}\right)^{2n}}}$ where f is the frequency and f_c is the cut-off frequency.

(c) The following signals are sampled at 16 kHz rate. Determine the first 4 samples of each signal.

(i) $x_1(t) = 5e^{-5000t}u(t)$ *(2 marks)*

(ii) $x_2(t) = 5\cos(3000\pi t)u(t)$ *(2 marks)*

(iii) $x_3(t) = 5e^{-5000t}\cos(3000\pi t)u(t)$ $x_3(t) = 5e^{-5000t}\cos(3000\pi t)u(t)$ *(2 marks)*

QUESTION 2 ***(25 marks)***

(a) (i) Write the formula that defines the DFT of a sequence $x[n]$. *(1 mark)*

(ii) Write the same formula in terms of twiddle factors. *(1 mark)*

(iii) Using your formula, determine the DFT of the sequence $x[n] = [1,\ 2,\ -3,\ 4]$. *(4 marks)*

(iv) Sketch the magnitude and phase spectrum of the sequence given in (iii) assuming that the sampling frequency is 200 Hz. *(4 marks)*

(b) (i) What is the purpose of using an FFT algorithm to evaluate the DFT? *(1 mark)*

(ii) Using a decimation-in-time FFT algorithm, evaluate the DFT of the sequence $x[n] = [1,\ 2,\ 0,\ -2,\ 2,\ 0,\ 3,\ -2]$ *(14 marks)*

QUESTION 3 ***(25 marks)***

(a) A discrete system in described by the difference equation

$$y(n) = 0.7y(n-1) - 0.1y(n-2) + x(n) + x(n-1)$$

(i) Find the zeros and poles of the system. *(5 marks)*

(ii) Is the system stable or not? Give a reason for your answer. *(2 marks)*

(iii) Obtain the impulse response of the system. *(6 marks)*

(b) For the discrete system with a transfer function $H(z) = \frac{k(z+1)}{(z-0.8)}$.

(i) Write down an expression for its frequency response. *(2 marks)*

(ii) Obtain a simplified expression for its frequency response at $\frac{1}{3}$ of the sampling frequency. *(6 marks)*

(iii) Determine the value of k if the magnitude of the transfer function at $\frac{1}{3}$ of the sampling frequency is 10. *(2 marks)*

(iv) Find the phase response in degrees of the system at $\frac{1}{3}$ of the sampling frequency. *(2 marks)*

QUESTION 4 ***(25 marks)***

(a) An IIR digital filter is described by the difference equation

$$y(n) = 0.8y(n-1) - 0.2y(n-3) + 0.3y(n-4) + x(n) + 3x(n-1) + 2x(n-2) + 4x(n-3)$$

Sketch a realization structure for this filter. *(5 marks)*

(b) An FIR has a transfer function given by $H(z) = 1 + 0.6z^{-1} + z^{-2}$. Given that the sampling rate is 7 kHz, determine the input signal frequency which will be maximally attenuated when passed through the filter. *(8 marks)*

(c) A DSP system has the transfer function $H(z) = \dfrac{1+z}{z-0.6}$.

Determine:

(i) The **unit step** response *y(n)* of the system. *(6 marks)*

(iii) The response *y(n)* of the system to an input sequence $x(n) = (0.4)^n u(n)$. *(6 marks)*

QUESTION 5 ***(25 marks)***

(a) Examine the stability or otherwise of the discrete system with the transfer function

$$H(z)=\frac{z+0.6}{(z-0.4)(z^2+\sqrt{2}\,z+1)}$$ *(6 marks)*

(b) A linear-phase FIR filter is to be designed with the following specifications:

Filter length, $N=5$

Normalized cut-off frequency = 0.55π rad

Window to be applied = Hamming

(i) Calculate the filter coefficients with accuracy of 4 decimal places. *(14 marks)*

(ii) Calculate the magnitude and phase response at a normalized frequency of $\frac{\pi}{4}$.

(5 marks)

TABLE OF Z-TRANSFORMS OF SOME COMMON SEQUENCES

Discrete-time sequence $x(n),\ n \geq 0$	**Z-transform** $H(z)$
$k\delta(n)$	k
k	$\frac{kz}{z-1}$
$ke^{-\alpha n}$	$\frac{kz}{z-e^{-\alpha}}$
$k\alpha^n$	$\frac{kz}{z-\alpha}$
kn	$\frac{kz}{(z-1)^2}$
kn^2	$\frac{kz(z+1)}{(z-1)^3}$
$kn\alpha^n$	$\frac{k\alpha z}{(z-\alpha)^2}$

QUANTIZATION

For a sine wave $SQNR = 6.02B + 1.76$ dB.

LOW PASS TO LOW PASS TRANSFORMATION

$s = \frac{s}{\omega_p'}$ where pre-warped frequency $\omega_p' = \tan\left(\frac{\pi f_c}{f_s}\right)$

SUMMARY OF IMPORTANT FEATURES OF SELECTED WINDOW FUNCTIONS

Name of Widow	Normalized Transition Width	Passband Ripple (dB)	Main lobe relative to Sidelobe (dB)	Max. Stopband attenuation (dB)	6 dB normalized bandwidth (bins)	Window Function $\omega(n)$, $\|n\| \le (N-1)/2$
Rectangular	0.9/N	0.7416	13	21	1.21	1
Hanning	3.1/N	0.0546	31	44	2.00	$0.5+0.5\cos\left(\frac{2\pi n}{N-1}\right)$
Hamming	3.3/N	0.0194	41	53	1.81	$0.54+0.46\cos\left(\frac{2\pi n}{N-1}\right)$
Blackman	5.5/N	0.0017	57	74	2.35	$0.42+0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$
Kaiser	2.93/N (β=4.54)	0.0274		50		$\frac{I_o\left(\beta\left\{1-\left[\frac{2n}{N-1}\right]^2\right\}^{\frac{1}{2}}\right)}{I_o(\beta)}$
	4.32/N (β=6.76	0.00275		70		
	5.71/N (β=8.96)	0.000275		90		

$$\text{Bin width} = \frac{f_s}{N} \text{ Hz}$$