University of Swaziland
Faculty of Science and Engineering
Department of Electrical and Electronic Engineering

Main Examination 2013

Title of Paper:	Signals and Systems I
Course Number:	EE331
Time Allowed:	3 hrs

Instructions:

1. Answer any four (4) questions.
2. Each question carries 25 marks.
3. Useful tables are attached at the end of the question paper

This paper should not be opened until permission has been given by the invigilator.

This paper contains eight (8) pages including this page.

Question 1

a) Define what a system is. [2]

b) Consider the periodic signal $x(t)$ shown in Fig. 1(b): [5]

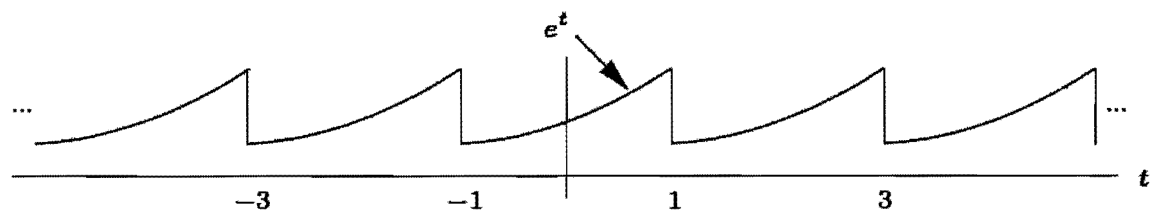


Fig. 1(b)

Find the closed-form expression for the Fourier series coefficients of $x(t)$.

c) Given that $e^{-4t} \xleftrightarrow{L} F(s)$. Find the inverse Laplace transforms of:

i) $F\left(\frac{s}{3}\right)$ [2]

ii) $F(s-2)+F(s+3)$ [2]

d) Find and sketch the first derivative of the following signal: [4]

$$x(t)=\operatorname{sgn}(t)=\begin{cases} 1, & t>0 \\ -1, & t<0 \end{cases}$$

e) Find the transfer function of the system determined by the input/ output relationship

$$\frac{d^2y}{dt^2}+7\frac{dy}{dt}+12y=\frac{dx}{dt}+2x$$

and determine its impulse response. [10]

Question 2

a) Determine the total impulse response $h(t)$ of the connected system in Fig. 2(a) [2]

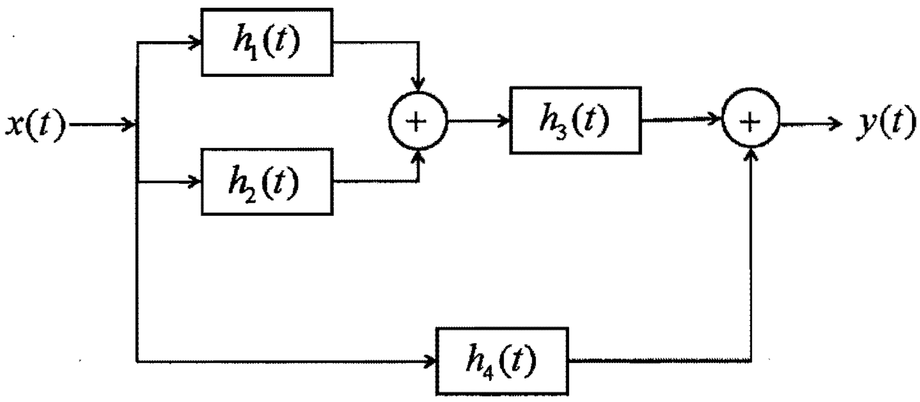


Fig. 2(a)

b) Using the graphical method, compute, $y(t) = x(t) * h(t)$ for $x(t) = 2[u(t) - u(t-1)]$ and $h(t) = e^{t}[u(t) - u(t-2)]$. [10]

c) Consider the periodic signal $x(t)$ given by the expression:

$$x(t) = (2+2j)e^{-j3t} - 3je^{-j2t} + 5 + 3je^{j2t} + (2-2j)e^{j3t}$$

What is the power of $x(t)$? Hint: Use the equation that relates the Fourier coefficients to the power of the signal. [3]

d) The causal LTI system S has the block diagram representation shown in Fig. 2(d). Determine a differential equation relating the input $x(t)$ to the output $y(t)$ of this system. [10]

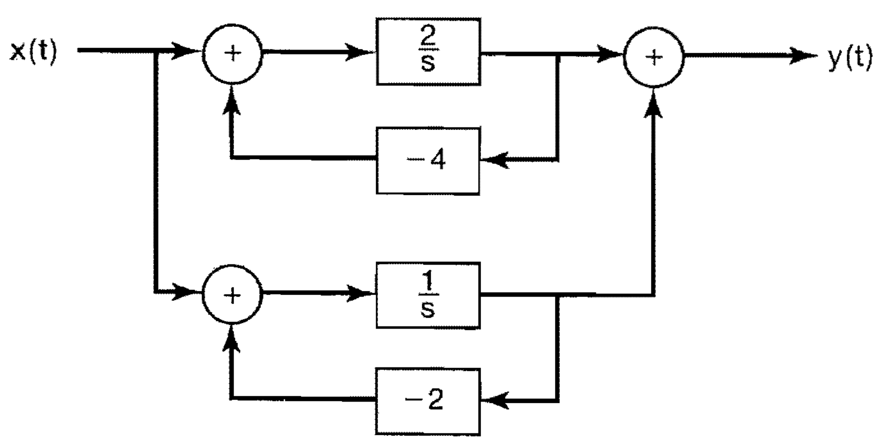


Fig. 2(d)

Question 3

a) State whether each of the following statements are TRUE or FALSE.

i) If a signal $f(t)$ is odd then $-f(-t)$ is an even signal. [1]

ii) A time-invariant system must be also linear. [1]

iii) The signal $x(t)=\cos\left(\sqrt{2}\pi t\right)+\sin\left(2\sqrt{2}\pi t\right)$ is periodic. [1]

iv) Periodic signals are always finite energy signals. [1]

b) Find the initial and final values of $y(t)$ if $Y(s)$ is given by: [4]

$$Y(s)=\frac{10(2s+3)}{s\left(s^2+2s+5\right)}$$

c) What are Walsh functions? [2]

d) Find and sketch the Fourier Series coefficients of the signal in Fig. 3(d). [8]

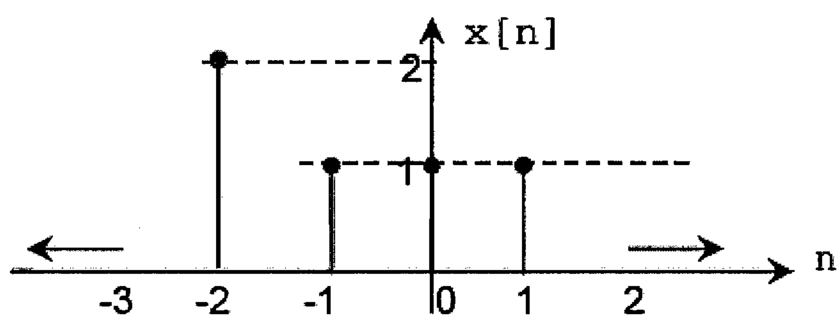


Fig. 3(d)

e) For the following signal, determine the location of the poles [5]

$$x(t)=\left[e^{2t}+3te^{2t}+e^{-t}\sin\left(6t\right)\right]u(t)$$

f) Differentiate between a stable and an unstable system. [2]

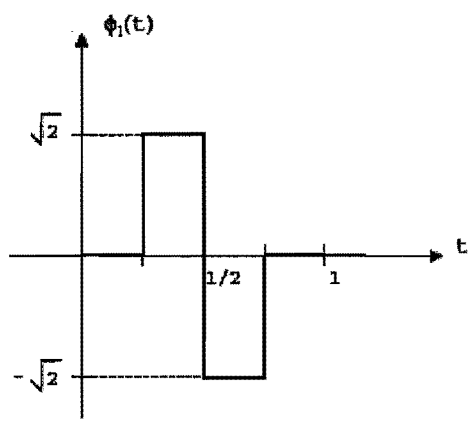
Question 4

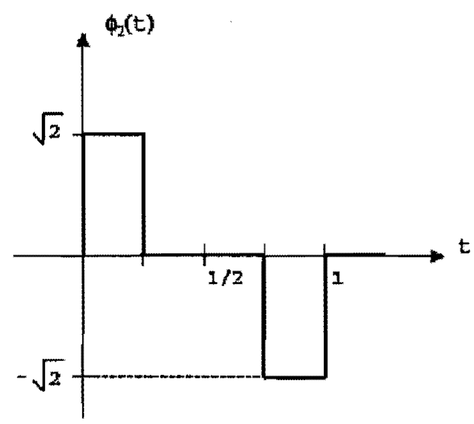
a) Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period.

i) $x[n] = \cos^2\left(\frac{\pi}{8}n\right)$ [2]

ii) $x(t) = e^{j\left[\frac{\pi t}{2} - 1\right]}$ [2]

b) Is the following pair of signals orthonormal over the interval (0,1)? Justify your answer? [5]





c) Sketch the even and the odd components of the signal in Fig. 4(c). [6]

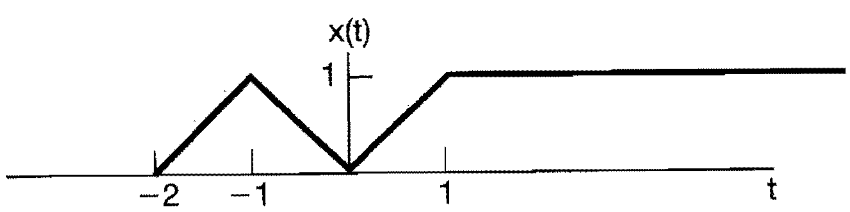


Fig. 4(c)

d) Using Laplace transforms solve the second-order linear differential equation [10]

$$y''(t) + 5y'(t) + 6y(t) = x(t)$$

with initial conditions $y(0) = 2$, $y'(0) = 1$ and $x(t) = e^{-t}u(t)$.

Question 5

a) Suppose $x(t)$ is shown in Fig. 5(a). Sketch the following signal, $[x(t)+x(-t)]u(t)$. [5]

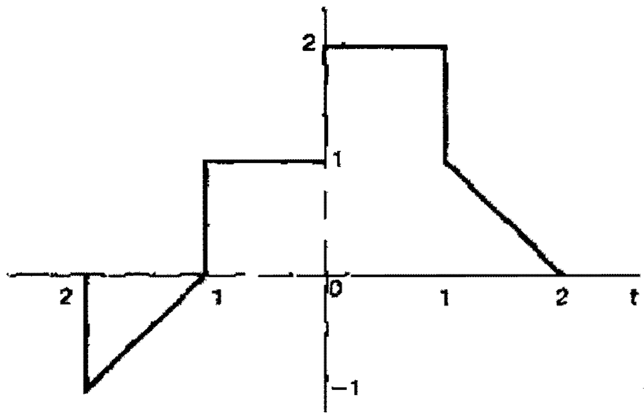


Fig. 5(a)

b) Given circuit in Fig. 5(b) with the elements shown.

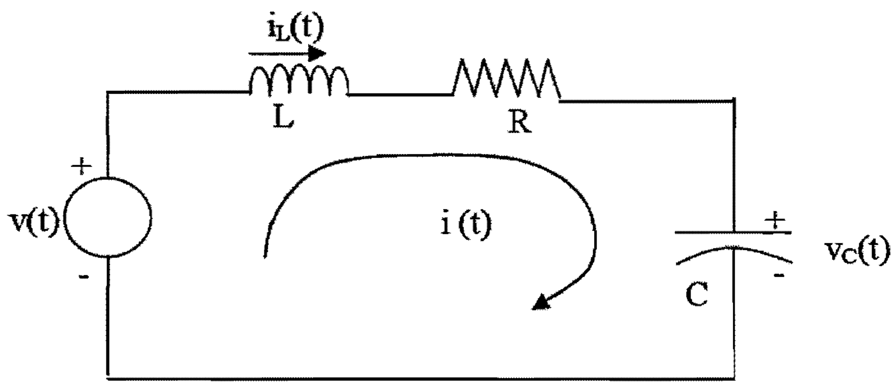


Fig. 5(b)

$L = 1H$, $R = 3\Omega$, $C = 0.5F$

With $v(t) = 10e^{-3t}u(t)V$, $i_L(0^-) = 0A$, $v_C(0^-) = 5V$

Find the loop current $i(t)$. [16]

c) Determine the region of convergence of the following function. [4]

$$x(t) = |t|e^{-2|t|}$$

Table of Laplace Transforms

delta function	$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1
shifted delta function	$\delta(t-a)$	$\xleftrightarrow{\mathcal{L}}$	e^{-as}
unit step	$u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$
ramp	$tu(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2}{s^3}$
n-th power	t^n	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\|t\|}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\xleftrightarrow{\mathcal{L}}$	$-F'(s)$
frequency n-th differentiation	$t^n f(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^n F^{(n)}(s)$
time differentiation	$f'(t) = \frac{d}{dt}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s) - f(0)$
time 2nd differentiation	$f''(t) = \frac{d^2}{dt^2}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2F(s) - sf(0) - f'(0)$
time n-th differentiation	$f^{(n)}(t) = \frac{d^n}{dt^n}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau = (u * f)(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)}{s^n} + \frac{f^{-1}(0)}{s^n} + \frac{f^{-2}(0)}{s^{n-1}} + \ldots + \frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

i) Time-shift (delay): $f(t-t_0) \overset{L}{\longleftrightarrow} F(s)e^{-st_0}, \quad t_0 > 0$

ii) Time differentiation: $\frac{df(t)}{dt} \overset{L}{\longleftrightarrow} sF(s) - f(0)$

iii) Time integration: $\int_0^t f(t)dt \overset{L}{\longleftrightarrow} \frac{F(s)}{s}$

iv) Linearity: $af(t) + bg(t) \overset{L}{\longleftrightarrow} aF(s) + bF(s)$

v) Convolution Integral: $x(t) * h(t) \overset{L}{\longleftrightarrow} X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t} f(t) \overset{L}{\longleftrightarrow} F(s-\alpha)$

vii) Multiplying by t: $tf(t) \overset{L}{\longleftrightarrow} -\frac{dF(s)}{ds}$

viii) Scaling: $f(at) \overset{L}{\longleftrightarrow} \frac{1}{a} F\left(\frac{s}{a}\right), \quad a > 0$

ix) Initial Value Theorem: $\lim_{s\to\infty}\{sF(s)\} = f(0)$

x) Final Value Theorem: $\lim_{s\to 0}\{sF(s)\} = f(\infty)$

Standard Table of Forced Response or Particular Solutions

	Input	Particular Solution
1	$cx^m(t)$	$a_0 + a_1x(t) + \ldots + a_mx^m(t)$
2	$cx^m(t)e^{ax(t)}$	$\left(a_0 + a_1x(t) + \ldots + a_mx^m(t)\right)e^{ax(t)}$
3	$cx^m(t)\cos\left(bx(t)\right)$	$\left(a_0 + a_1x(t) + \ldots + a_mx^m(t)\right)\cos\left(bx(t)\right) + \left(c_0 + c_1x(t) + \ldots + c_mx^m(t)\right)\sin\left(bx(t)\right)$
4	$cx^m(t)\sin\left(bx(t)\right)$	$\left(a_0 + a_1x(t) + \ldots + a_mx^m(t)\right)\sin\left(bx(t)\right) + \left(c_0 + c_1x(t) + \ldots + c_mx^m(t)\right)\cos\left(bx(t)\right)$

where $c, a_0, a_1, a_m, c_0, c_1, c_m$ are constants.