University of Swaziland
Faculty of Science and Engineering
Department of Electrical and Electronic Engineering

Supplementary Examination 2014

Title of Paper:	Signals and Systems I
Course Number:	EE331
Time Allowed:	3 hrs

Instructions:

1. Answer any four (4) questions.
2. Each question carries 25 marks.
3. Useful tables are attached at the end of the question paper

This paper should not be opened until permission has been given by the invigilator.

This paper contains nine (9) pages including this page.

Question 1

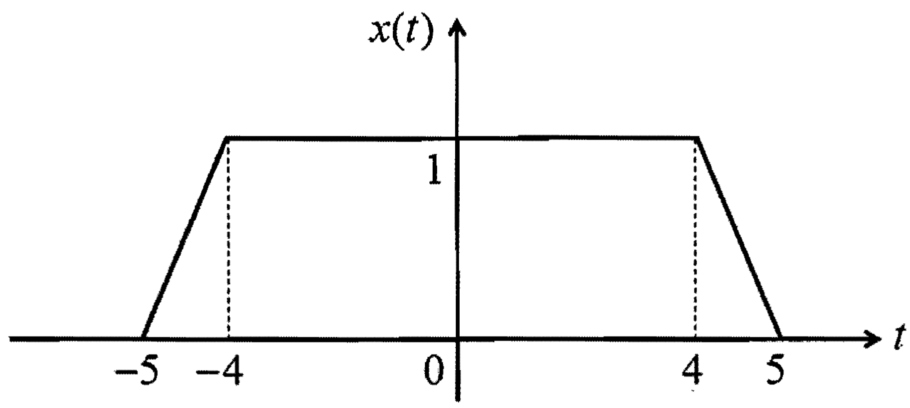
a) Find the inverse Laplace transform of the following function: [8]

$$\frac{s+4}{s\left(s^2+5s+6\right)}$$

b) Determine whether the following signals are energy signals or power signals and calculate their energy or power.

i) $x[n]=\left(\frac{1}{2}\right)^n u[n]$ [3]

ii)



[5]

c) Find the Fourier series coefficients of the signal: [5]

$$x(t)=\sin^3(3\pi t)$$

d) State whether each of the following statements are TRUE or FALSE.

i) If a signal $f(t)$ is odd then $-f(-t)$ is an even signal. [1]
ii) A time-invariant system must be also linear. [1]
iii) The signal $x(t)=\cos\left(\sqrt{2}\pi t\right)+\sin\left(2\sqrt{2}\pi t\right)$ is periodic. [1]
iv) Periodic signals are always finite energy signals. [1]

Question 2

a) Find and sketch the first derivative of the following signal: [4]

$$x(t) = \text{sgn}(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$$

b) Find the transfer function of the system determined by the input/ output relationship

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 12y = \frac{dx}{dt} + 2x$$

and determine its impulse response. [10]

c) Find the initial and final values of $y(t)$ if $Y(s)$ is given by: [4]

$$Y(s) = \frac{10(2s+3)}{s\left(s^2+2s+5\right)}$$

d) From the given signal $x(t)$ as shown in Fig. 2(d), plot $x\left(1-\frac{t}{2}\right)$ [3]

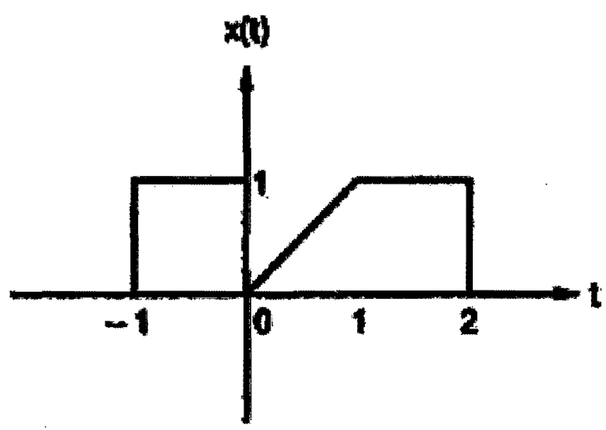


Fig. 2(d)

e) Find and sketch the region of convergence (ROC) for the signal in Fig. 2(e). [4]

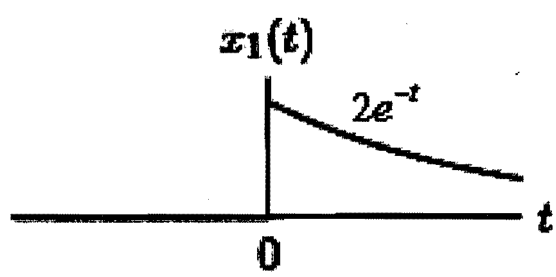


Fig. 2(e)

Question 3

a) Find and sketch the Fourier Series coefficients of the signal in Fig. 3(a). [8]

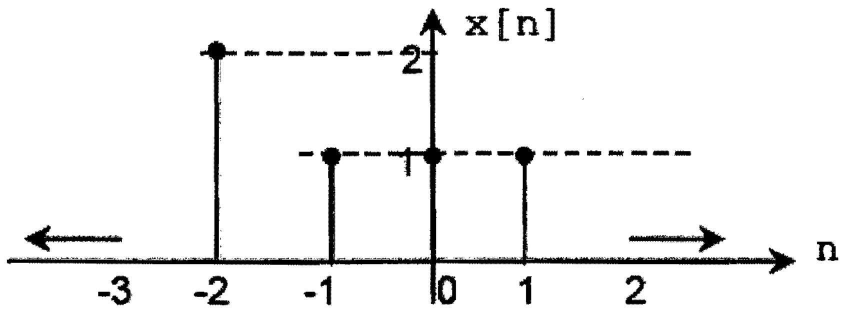


Fig. 3(a)

b) Find the fundamental period (in seconds) of the sum signal: [3]

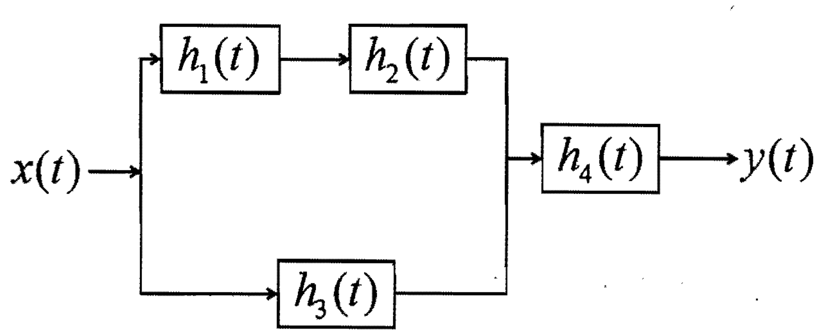
$$x(t) = 3\sin\left(\frac{\pi t}{3}\right) + 5\cos\left(\frac{\pi t}{5}\right) + 7\sin\left(\frac{\pi t}{7}\right)$$

c) For $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$, compute $y[n] = x[n] * h[n]$. [3]

d) Find the even and odd components of the following: [4]

$$x(t) = \cos^2\left(\frac{\pi t}{2}\right)$$

e) Determine the overall impulse response of the following LTI system: [2]



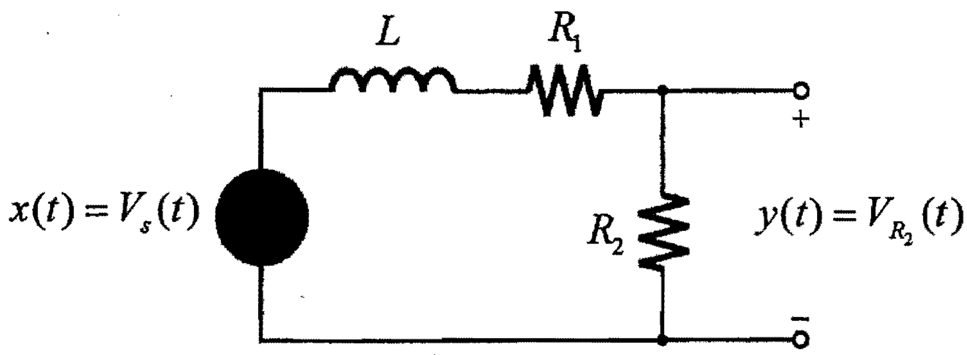
f) For the following signal, determine the location of the poles [5]

$$x(t) = \left[e^{2t} + 3te^{2t} + e^{-t}\sin(6t)\right]u(t)$$

Question 4

a) Using the graphical method, compute, $y(t) = x(t) * h(t)$ for $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t}u(t)$. [6]

b) Derive the differential equation in terms of $x(t)$ and $y(t)$ that relates all the circuit components in the circuit below: [10]



c) Determine if each of the following systems is invertible. If it is, find the associated inverse systems.

i) $y(t) = x(t-4)$ [2]

ii) $y[n] = x[2n]$ [2]

d) State the three Dirichlet conditions. [3]

e) What are Walsh functions? [2]

Question 5

a) Determine the forced response of the system:

$$5\frac{dy(t)}{dt}+10y(t)=2x(t)$$

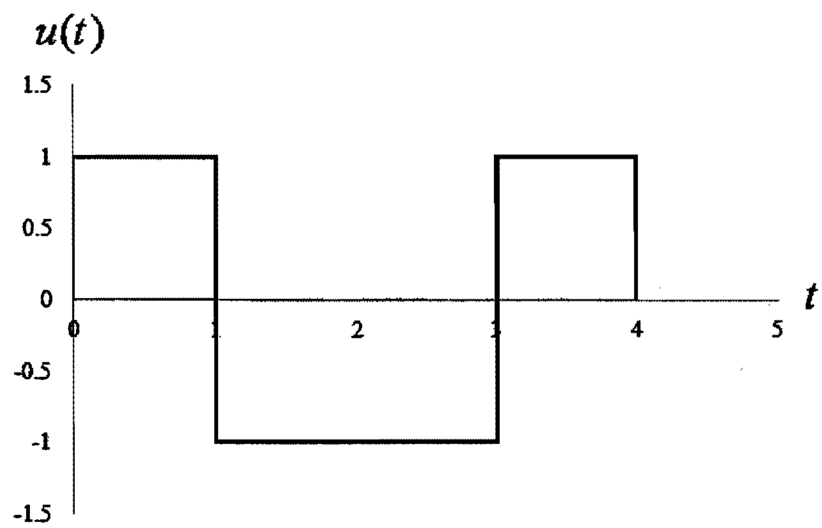
for the input $x(t)=2u(t)$, assume zero initial conditions. [10]

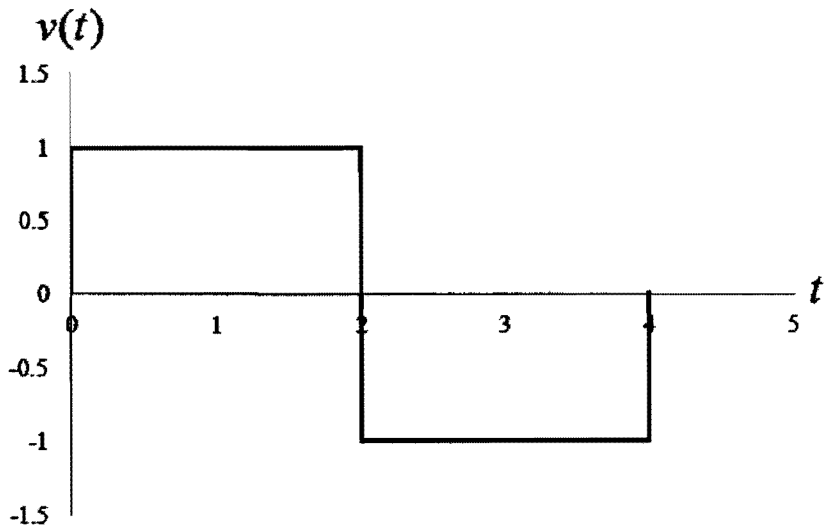
N. B. Use the Standard Table of Forced Response or Particular Solutions to solve this.

b) Draw a block diagram representation for the causal LTI system described by the following difference equation: [3]

$$y[n]=\frac{1}{3}y[n-1]+x[n-1]$$

c) Is the following pair of signals orthogonal over the interval (0,4)? Prove your answer. [4]





d) For the following signal find an expression for its first derivative: [2]

$$x(t) = 2a(t) - 3r(t-1) - 4u(t-3)$$

e) Differentiate between a deterministic and a probabilistic signals and give examples of each. [4]

f) Define what a signal is. [2]

Table of Laplace Transforms

delta function	$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1
shifted delta function	$\delta(t-a)$	$\xleftrightarrow{\mathcal{L}}$	e^{-as}
unit step	$u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$
ramp	$tu(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2}{s^3}$
n-th power	t^n	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\lvert t\rvert}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\xleftrightarrow{\mathcal{L}}$	$-F'(s)$
frequency n-th differentiation	$t^n f(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^n F^{(n)}(s)$
time differentiation	$f'(t) = \frac{d}{dt}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s) - f(0)$
time 2nd differentiation	$f''(t) = \frac{d^2}{dt^2}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2F(s) - sf(0) - f'(0)$
time n-th differentiation	$f^{(n)}(t) = \frac{d^n}{dt^n}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau = (u * f)(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)}{s^n} + \frac{f^{-1}(0)}{s^n} + \frac{f^{-2}(0)}{s^{n-1}} + \ldots + \frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

i) Time-shift (delay): $f(t-t_0)\xleftrightarrow{L}F(s)e^{-st_0},\ \ t_0>0$

ii) Time differentiation: $\frac{df(t)}{dt}\xleftrightarrow{L}sF(s)-f(0)$

iii) Time integration: $\int_0^t f(t)dt\xleftrightarrow{L}\frac{F(s)}{s}$

iv) Linearity: $af(t)+bg(t)\xleftrightarrow{L}aF(s)+bF(s)$

v) Convolution Integral: $x(t)*h(t)\xleftrightarrow{L}X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t}f(t)\xleftrightarrow{L}F(s-\alpha)$

vii) Multiplying by t: $tf(t)\xleftrightarrow{L}-\frac{dF(s)}{ds}$

viii) Scaling: $f(at)\xleftrightarrow{L}\frac{1}{a}F\left(\frac{s}{a}\right),\ \ a>0$

ix) Initial Value Theorem: $\lim_{s\to\infty}\{sF(s)\}=f(0)$

x) Final Value Theorem: $\lim_{s\to 0}\{sF(s)\}=f(\infty)$

Standard Table of Forced Response or Particular Solutions

	Input	Particular Solution
1	$cx^m(t)$	$a_0+a_1x(t)+\ldots+a_mx^m(t)$
2	$cx^m(t)e^{ax(t)}$	$\left(a_0+a_1x(t)+\ldots+a_mx^m(t)\right)e^{ax(t)}$
3	$cx^m(t)\cos\left(bx(t)\right)$	$\left(a_0+a_1x(t)+\ldots+a_mx^m(t)\right)\cos\left(bx(t)\right)+\left(c_0+c_1x(t)+\ldots+c_mx^m(t)\right)\sin\left(bx(t)\right)$
4	$cx^m(t)\sin\left(bx(t)\right)$	$\left(a_0+a_1x(t)+\ldots+a_mx^m(t)\right)\sin\left(bx(t)\right)+\left(c_0+c_1x(t)+\ldots+c_mx^m(t)\right)\cos\left(bx(t)\right)$

where $c, a_0, a_1, a_m, c_0, c_1, c_m$ are constants.