# UNIVERSITY OF SWAZILAND 

FACULTY OF SCIENCE<br>DEPARTMENT OF ELECTRONIC ENGINEERING

## MAIN EXAMINATION 2013

## Title of the Paper: <br> Electromagnetic Fields I

Course Number: EE341
Time Allowed: Three Hours.

Instructions:

1. To answer, pick any questions to sum a total of $100 \%$
from 14 questions in the following pages.
2. The answers should be neatly written in the space provided in the question book. Use the answer book as a scratch pad.
3. This paper has 9 pages, including this page.

DO NOT OPEN THE PAPER
UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Q1: Given a scalar function $f(x, y, z)=x \cdot y$, find (i) $\int f \cdot d \vec{l}$ and (ii) $\int f \cdot d l$ along a straight line from $(0,0,0)$ to ( $1,1,0$ ). $\quad 10$ pts ( 5 pts for each (i) and (ii))

Q2: Given a scalar function, $h(x, y)=\left(x^{2}+y^{2}\right) / 2 x$, the height of a slanted cone shown in Fig. Q2-1, (i) calculate graphically the maximum change (gradient) of the height at the location $P_{x}(6,6)$ and the direction of the change; (ii) calculate the same but analytically. Check if the two answers are close. $\quad 15$ pts ( 5 pts for (i) and 10 pts for (ii).)

$h(x, y)=\left(x^{2}+y^{2}\right) / 2 x$ $h$-axis out of the paper contour (constant height, "h") Fig. Q2-1 of a slant cone.

Q3: Given two field patterns shown in Fig. Q3-1 and -2, (i) by inspection determine and mark the area which has curl $1 \neq 0$ or $\operatorname{div} \neq 0$ or both $\neq 0$ of the pattern. Then (ii) analytically calculate the non-zero curl or divergence to prove. Take closed surface anywhere in the pattern but must be specified. The fields are in xy-plane only, no contribution in z -axis top and bottom. The closed surface may be cubically or cylindrically bounded. $\quad 10 \mathrm{pts}$ ( 5 pts for each pattern.)
$\mathbf{A}=\hat{\mathbf{x}} x y^{2}$, for $-10 \leq x, y \leq 10$


Fig. Q3-1
$\mathbf{A}=\hat{\mathbf{r}} r^{2}+\hat{\phi} r^{2} \sin \phi$, for $\left\{\begin{array}{l}0 \leq r \leq 10 \\ 0 \leq \phi \leq 2 \pi\end{array}\right.$


Fig. Q3-2

Q4: List any five pairs of dual equation in electromagnetic fields. $\quad 10 \mathrm{pts}$ ( 2 pts for each pair)

| term | Electric Fields | Magnetic Fields |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

Q5: Under static condition, show (i) which equation, which listed in the table of dual EM equations will degenerate into Kirchhoff's Voltage Law, specifying the necessary conditions; (ii) which will degenerate into Kirchhoff's Current Law likewise. Current has the same features as B field. $\quad 10$ pts ( 5 pts for each)

Q6:An infinitely long line charge with a line density $+\mathrm{q}_{1} \mathrm{Coul} / \mathrm{Mtr}$ is located d Mtr above an infinitive perfect conducting plane. (i) Find the charge density on the plane. Use the image method. (ii) Is there any dual method in static magnetic fields and give the reason behind? $\quad 10 \mathrm{pts}$ ( 6 pts for the first question, 4 pts for the second).


Fig. Q6-1

Q7: A coaxial cable has a inner radius $r_{i}$ and outer radius $r_{0}$ with insulation material $\varepsilon / \mu_{0}$. Consider no end fringing effects. (i) Calculate the cable per unit inductance and capacitance. (ii) the Characteristic impedance $\mathrm{z}_{\mathrm{o}}$. $\quad 10 \mathrm{pts}$ ( 4 pts for each answer in (i) and 2 pts for (ii))

Q8: An electric dipole antenna has a dipole moment $1 / 9$ coul-mtr and its direction is oriented in the z-axis. Calculate (i) the electric field at 1 KM away with $\theta=0^{\circ}$ and (ii) the same with $\theta=90^{\circ}$. (iii) Comment on the direction of the two fields with respect to the dipole orientation. 15 pts ( 5 pts for each.)


Fig. Q8-1

Q9: Two infinitely long line charges carry equal and opposite charge density q , are located each at d meters away from z -axis in the cylindrical coordinates and in xz plane. (possibly a Cartesian). Find the zero potential surface. $\quad 10 \mathrm{pts}$

Q10: A coaxial cable has a inner radius $r_{i}$ and outer radius $r_{0}$ with insulation material $\varepsilon / \mu_{0}$. Consider no end fringing effects. (i) Find the total electric energy stored in this 1 meter long cable, energized by a source charge $\mathrm{q}_{1} \mathrm{Coul} / \mathrm{Mtr}$. (ii) Find the total magnetic energy stored in this 1 meter long cable, energized by a source current $\mathrm{I}_{\mathrm{s}}$. $\quad \mathbf{1 0} \mathbf{~ p t s}$ ( 5 pts for each)

Q11: Two short parallel pieces of wire are parallel to the x -axis, separated 2 d mMtr apart from x -axis, lying on xz -plane The wires carry a current of I Amps in opposite direction and so each is stored an opposite charge of q coul on the wires. Dipole conditions are applicable. (i) Determine the $B$-field away from $y$-axis $(0, \infty)$; (ii) determine the E-field likewise. Notice any special point about these two fields.
 15 pts ( 5 pts each) (hint: application of dipole potential formula is, recommended)

Q12(i) Show that if no surface current densities exist at the parallel interfaces shown in Fig. Q12-1, the relationship between $\theta_{4}$ and $\theta_{1}$ is independent of $\mu_{2}$. (ii) Show the same for independent of $\varepsilon_{2}$ for electric fields if no surface charge densities exist likewise. $\quad \mathbf{1 0}$ pts ( 5 pts for each)


Q13: A current coil of radius $r_{0}$ carries a current $I$. Determine the vector potential of this coil at the point on its axis and z meters away from the coil plane. 10pts


Q14: A magnetic circuit with all the pertinent dimensions in centimeters is shown in Fig. Q14-1. Determine the current in the 1600 -turn coil to establish a flux density of 0.75 T in each air gap. Given H (in $\mathrm{At} / \mathrm{m}$ )/B (in T ) $=1000$. (hint: using analogy of Ohm's law in magnetic circuit) 10 pts


Fig. Q14-1

