University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Main Examination 2014

Title of Paper	:	**Introduction to Digital Signal Processing**
Course Number	:	**EE443**
Time Allowed	:	**3 hrs**
Instructions	:	

1. **Answer any four (4) questions**
2. **Each question carries 25 marks**
3. **Useful information is attached at the end of the question paper**

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The paper consists of six (6) pages

QUESTION 1

(a) Describe the digital signal processing scheme (DSP) and explain the function of each block? (10 marks)

(b) A digital signal processing (DSP) system is described by the difference equation

$$y(n) - 0.5y(n-1) = 5(0.2)^n u(n)$$

Determine the solution when the initial condition is given by $y(-1) = 1$.

(10 marks)

(c) Find the z-transform of the following

$$X(z) = \frac{10z}{z^2 - z + 1}$$

(5 marks)

QUESTION 2

(a) A relaxed (zero initial conditions) DSP system is described by the difference equation

$$y(n) + 0.1\,y(n-1) - 0.2\,y(n-2) = x(n) + x(n-1)$$

(i) Determine the impulse response $y(n)$ due to the impulse sequence $x(n) = \delta(n)$ (7 marks)

(ii) Determine the system response $y(n)$ due to the unit step function excitation, where $u(n) = 1 \ for\, n \geq 0$. (7 marks)

(b) Given a sequence $x(n)$ for $0 \leq n \leq 3$, where $x(0) = 1, x(1) = 2, x(2) = 3, x(3) = 4$. Evaluate its DFT $X(k)$ (6 marks)

(c) List any applications for digital signal processing? (5 marks)

QUESTION 3

(a)

(i) Calculate the filter coefficients for a 3-tap FIR lowpass filter with a cutoff frequency of 800 Hz and a sampling rate of 8,000 Hz using the Hamming window method. (6 marks)

(ii) Determine the transfer function and difference equation of the designed FIR system (4 marks)

(iii) Determine the magnitude frequency response and phase for $\Omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ $and\ \pi\ radians$ (5 marks)

(b) Find the z-transform of the sequence defined by

$$x(n) = u(n) - (0.4)^n u(n)$$ (2 marks)

(c) Determine the z-transform of

$$y(n) = (0.5)^{(n-1)}\,u(n-5),$$

Where $u(n-5) = 1\ for\ n \geq 5\ and\ u(n) = 0\ for\ n < 5$ (3 marks)

(d) Compare the Von Neumann and the Harvard architecture? (5 marks)

QUESTION 4

(a) Given a sequence $x(n)$ for $0 \leq n \leq 3$, where $x(0) = 1, x(1) = 2, x(2) = 3, x(3) = 4$.

(i) Evaluate its DFT $X(k)$ using the decimation-in-frequency FFT method (5 marks)

(ii) Determine the number of complex multiplications (2 marks)

(b) Compare the executions cycle of the two architectures, Von Neumann and the Harvard architecture? (10 marks)

(c) Given two sequences,

$$x_1(n) = 3\delta(n) + 2\delta(n-1)$$
$$x_2(n) = \delta(n-1) + 2\delta(n)$$

(i) Find the z-transform of their convolution $X(z) = Z(x_1(n) * x_2(n))$ (4 marks)

(ii) Determine the convolution sum using the z-transform (4 marks)

QUESTION 5

(a) Given a second-order transfer function

$$H(z) = \frac{0.5(1 - z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-2}},$$

Perform the filter realizations and write the difference equations using the following realizations:

(i) Direct form I and direct form II (10 marks)
(ii) Cascade form via the first –order sections (10 marks)

(b)A discrete-time processing operation is defined by the recurrence equation

$$y[n] = x[n] + 2x[n-1] + 3x[n-3]$$

What is the unit-sample response $h[n]$ of the processor? (5 Marks)

Table 1: Properties of z-transform

Property	Time Domain	z-Transform
Linearity	$ax_1(n) + bx_2(n)$	$aZ(x_1(n)) + bZ(x_2(n))$
Shift theorem	$x(n-m)$	$z^{-m}X(z)$
Linear convolution	$x_1(n)*x_2(n) = \sum_{k=0}^{\infty} x_1(n-k)x_2(k)$	$X_1(z)X_2(z)$

Table 2: Partial fraction(s) and formulas for constant(s).

Partial fraction with the first-order real pole:	
$\frac{R}{z-p}$	$R = (z-p)\frac{X(z)}{z}\Big\vert_{z=p}$
Partial fraction with the first-order complex poles:	
$\frac{Az}{(z-P)} + \frac{A^*z}{(z-P^*)}$	$A = (z-P)\frac{X(z)}{z}\Big\vert_{z=P}$
P^* = complex conjugate of P	
A^* = complex conjugate of A	
Partial fraction with mth-order real poles:	
$\frac{R_m}{(z-p)} + \frac{R_{m-1}}{(z-p)^2} + \cdots + \frac{R_1}{(z-p)^m}$	$R_k = \frac{1}{(k-1)!}\frac{d^{k-1}}{dz^{k-1}}\left((z-p)^m \frac{X(z)}{z}\right)\Big\vert_{z=p}$

Table 3: Summary of ideal impulse r esponses for standard FIR filters.

Filter Type	Ideal Impulse Response $h(n)$ (noncausal FIR coefficients)
Lowpass:	$h(n) = \begin{cases} \frac{\Omega_c}{\pi} & n = 0 \\ \frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Highpass:	$h(n) = \begin{cases} \frac{\pi - \Omega_c}{\pi} & n = 0 \\ -\frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Bandpass:	$h(n) = \begin{cases} \frac{\Omega_H - \Omega_L}{\pi} & n = 0 \\ \frac{\sin(\Omega_H n)}{n\pi} - \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Bandstop:	$h(n) = \begin{cases} \frac{\pi - \Omega_H + \Omega_L}{\pi} & n = 0 \\ -\frac{\sin(\Omega_H n)}{n\pi} + \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$

Causal FIR filter coefficients: shifting $h(n)$ to the right by M samples.
Transfer function:

$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_{2M} z^{-2M}$
where $b_n = h(n-M)$, $n = 0, 1, \cdots, 2M$

The Z-transform

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$\|z\| > 0$
3	$au(n)$	$\dfrac{az}{z-1}$	$\|z\| > 1$
4	$nu(n)$	$\dfrac{z}{(z-1)^2}$	$\|z\| > 1$
5	$n^2u(n)$	$\dfrac{z(z+1)}{(z-1)^3}$	$\|z\| > 1$
6	$a^n u(n)$	$\dfrac{z}{z-a}$	$\|z\| > \|a\|$
7	$e^{-na}u(n)$	$\dfrac{z}{(z-e^{-a})}$	$\|z\| > e^{-a}$
8	$na^n u(n)$	$\dfrac{az}{(z-a)^2}$	$\|z\| > \|a\|$
9	$\sin(an)u(n)$	$\dfrac{z\sin(a)}{z^2 - 2z\cos(a) + 1}$	$\|z\| > 1$
10	$\cos(an)u(n)$	$\dfrac{z[z - \cos(a)]}{z^2 - 2z\cos(a) + 1}$	$\|z\| > 1$
11	$a^n \sin(bn)u(n)$	$\dfrac{[a\sin(b)]z}{z^2 - [2a\cos(b)]z + a^2}$	$\|z\| > \|a\|$
12	$a^n \cos(bn)u(n)$	$\dfrac{z[z - a\cos(b)]}{z^2 - [2a\cos(b)]z + a^{-2}}$	$\|z\| > \|a\|$
13	$e^{-an}\sin(bn)u(n)$	$\dfrac{[e^{-a}\sin(b)]z}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\|z\| > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\dfrac{z[z - e^{-a}\cos(b)]}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\|z\| > e^{-a}$
15	$2\|A\|\|P\|^n \cos(n\theta + \phi)u(n)$ where P and A are complex constants defined by $P = \|P\|\angle\theta, A = \|A\|\angle\phi$	$\dfrac{Az}{z-P} + \dfrac{A^* z}{z-P^*}$	