

**UNIVERSITY OF SWAZILAND**



**MAIN EXAMINATION PAPER 2014**

**TITLE OF PAPER : PROBABILITY AND STATISTICS**

**COURSE CODE : EE 301**

**TIME ALLOWED : THREE (3) HOURS**

**INSTRUCTIONS : ANSWER ANY FIVE QUESTIONS.**

**REQUIREMENTS : SCIENTIFIC CALCULATOR AND  
STATISTICAL TABLES.**

### Question 1

- a) Let A and B be events in a probability space. The probability of event A is  $P(A) = 1/3$  and the conditional probability of B given  $A^c$  is  $P(B|A^c) = 1/4$ . Find the probability that A or B occurs.  
(5 Marks)
- b) Let E and F be two events for which the probability that at least one of them occurs is  $3/4$ . Find the probability that neither E nor F occurs.  
(5 Marks)
- c) One tosses a fair coin twice. The two events of interest are:  $A = \{\text{first toss is a head}\}$  and  $B = \{\text{second toss is a head}\}$ . Are A and B independent? And are they disjoint?  
(5 Marks)
- d) In a certain country it is established that 0.5% of the population suffers from a certain disease. For this disease there exists a test that gives the correct diagnosis for 80% of healthy persons and for 98% of sick persons. A person is tested and found sick. Find the probability that the diagnosis is wrong, i.e. that the person is actually healthy.  
(5 Marks)

### Question 2

- a) On January 28, 1986 the space shuttle Challenger exploded about one minute after the launch. The cause of the disaster was explosion of the main fuel tank, caused by flames of hot gas erupting from one of the solid rocket boosters. These rocket boosters are manufactured in segments, joint together with O-rings. Each rocket booster has three O-rings and per launch two rocket boosters are used, so in total six O-rings each time. Based on data on the number of failed O-rings, available from previous launches, it was found that the probability  $p$  that an individual O-ring fails depends on the launch temperature  $t$  (in degrees Fahrenheit) according to

$$p = \frac{\exp(a + bt)}{1 + \exp(a + bt)}$$

with  $a = 5.085$  and  $b = -0.1156$ . Hence,  $p$  increases with decreasing launch temperature. At the time of the fatal launch of the Challenger,  $t$  was extremely low: 31 degrees Fahrenheit. Although the above formula is based on data for which  $t > 50$  degrees Fahrenheit, let us use this formula also for  $t = 31$  degrees Fahrenheit.

Find the probability of at least one O-ring failing during the 1986 Challenger launch.  
(8 Marks)

- b) A machine fastens plastic screw-on caps onto containers of motor oil. If the machine applies more torque than the cap can withstand, the cap will break. Both the applied torque and the strength of the caps vary. The capping machine torque is a normally distributed random variable with mean 0.79Nm and standard deviation 0.10Nm. The cap strength, being the torque that would break the cap, is also a normally distributed random variable with mean 1.13Nm and standard deviation 0.14Nm. Assume that the cap strength and the applied torque are independent.

i. What is the probability that a cap will break while being fastened by the capping machine?

(5 Marks)

ii. Let  $X$  be an exponentially distributed random variable with parameter  $\lambda = 1/5$ . Find the conditional probability  $P(x < 5 | 3 < x < 6)$ .

(7 Marks)

### Question 3

The random variable  $X$  is uniformly distributed on the interval  $(0, 1)$ . Derive the PDF of the random variable  $Y = -\ln X$ . Consider two independent random variables  $X_1$  and  $X_2$ , distributed exponentially with  $\lambda = 1$ . That is,

$$f_X(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the PDF of  $X_1 + X_2$ .

(20 Marks)

### Question 4

a) The random variable  $W$  is uniformly distributed on the interval  $(\pi, 2\pi)$ . What can you say about  $E[\sin(W)]$  and  $\sin[E(W)]$ ? Which one is bigger? And if they are equal, are they also equal to zero?

A pulse of light has energy  $X$  that is a random variable with parameter  $\lambda$ , i.e., its PDF is

$$f_X(x) = \lambda^2 x e^{-\lambda x}, \text{ for } x \geq 0.$$

(8 Marks)

b) This pulse illuminates an ideal photon-counting detector whose output  $N$  is a Poisson-distributed random variable with mean  $x$  when  $X = x$ , i.e., its conditional PMF is

$$p_{N|X}(n|x) = \frac{\lambda^n e^{-\lambda}}{n!}, \text{ for } n = 0, 1, 2, \dots$$

Find  $E[N]$  and  $\text{Var}[N]$

(12 Marks)

### Question 5

(a) Suppose that the discrete random variable  $X$  has the probability function

$$P(X = x) = (1 - \theta)^{x-1} \theta, \quad x = 1, 2, \dots$$

Show that X has moment generating function

$$M_X(t) = \frac{e^{t\theta}}{1 - e^t(1 - \theta)}, \quad t < -\ln(1 - \theta)$$

Hence show that the expected value of X is  $1/\theta$  and that the variance of X is  $(1 - \theta)\theta^{-2}$

(10 Marks)

(b) The random variable X has a Chi-squared distribution with  $k$  degrees of freedom ( $k = 1, 2, 3, \dots$ ), which has moment generating function (mgf)  $m(t) = (1 - 2t)^{-k/2}$  for  $t < 1/2$ .

i. Using the  $m(t)$ , find the mean and variance of X.

(5 Marks)

ii. In the case  $k = 4$ , the probability density function is given by

$$f(x) = \frac{1}{4} x e^{-x/2}, \quad x > 0$$

Using integration, confirm that the mgf of the Chi-squared distribution with 4 degrees of freedom is  $m(t) = (1 - 2t)^{-2}$  for  $t < 1/2$ .

(5 Marks)

### Question 6

Suppose two random variables X and Y have joint density function

$$f(x, y) = (x + y), \text{ if } 0 < x < 1, 0 < y < 1$$

Find the density function of X and the density function of Y.

(14 Marks)

Are X and Y independent? Why or why not?

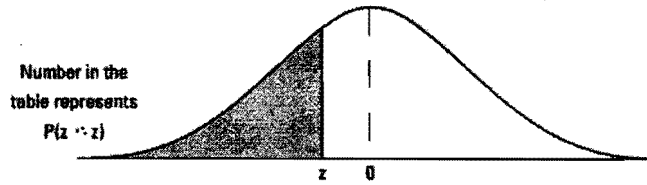
(6 Marks)

### Question 7

It is assumed that the IQ-score for the population of students in Electrical and Electronic Engineering Department is distributed as  $N(\mu, \sigma^2)$ , with  $\sigma = 10$ . The Faculty of Science and Engineering students' average of  $\mu = 110$  is the null hypothesis  $H_0$ . To test the alternative hypothesis,  $H_a : \mu = 120$ , that E & E students are smarter, a random sample is taken: the IQ of 16 E & E students is measured and the mean comes out as 114.5. Are they smarter? Use the Best test. What is the decision at the 10% level of significance, and what is the decision at the 5% level of significance?

(20 Marks)

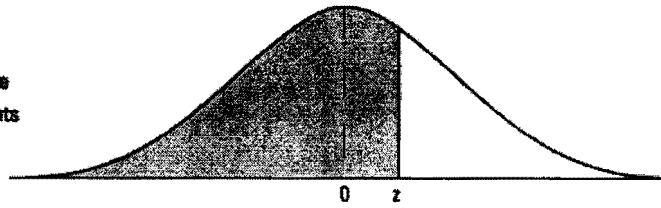
**Table A-2 The cdf of the Z Distribution (the Z Table)**



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.8	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0003	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0009	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

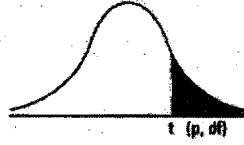
**Table A-2 (continued)**

Number in the  
table represents  
 $P(z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7548
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9308	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

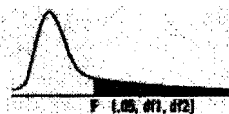
t-distribution showing area to the right



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
∞	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905

Table A-5

The F-Table



df2/df1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120
1	161.4478	199.5000	215.7073	224.5832	230.1619	233.9860	236.7684	238.8827	240.5433	241.8817	243.0060	245.9499	248.0131	249.0518	250.0861	251.1432	252.1957	253.252
2	18.5128	19.0000	19.1643	19.2468	19.2984	19.3295	19.3532	19.3716	19.3848	19.3959	19.4125	19.4291	19.4458	19.4541	19.4624	19.4707	19.4791	19.487
3	10.1280	9.5521	9.2768	9.1172	9.0126	8.9406	8.8887	8.8452	8.8123	8.7855	8.7448	8.7029	8.6682	8.6385	8.6106	8.5844	8.5720	8.549
4	7.7086	6.9443	6.5914	6.2882	6.2581	6.1631	6.0942	6.0410	5.9888	5.9544	5.9117	5.8578	5.8025	5.7744	5.7459	5.7170	5.6877	5.658
5	6.6079	5.7861	5.4085	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.398
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990	4.0600	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.704
7	5.5914	4.7374	4.3488	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.267
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.968
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.747
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.9130	2.8450	2.7740	2.7372	2.6996	2.6609	2.6211	2.580
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536	2.7876	2.7186	2.6464	2.6090	2.5705	2.5309	2.4901	2.448
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.5065	2.4668	2.4259	2.3842	2.341
13	4.6672	3.8054	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.6710	2.6037	2.5331	2.4589	2.4212	2.3803	2.3382	2.2956	2.252
14	4.6001	3.7383	3.3433	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022	2.5342	2.4630	2.3879	2.3497	2.3082	2.2654	2.2228	2.177
15	4.5431	3.6823	3.2874	3.0566	2.9013	2.7905	2.7066	2.6408	2.5878	2.5437	2.4753	2.4034	2.3275	2.2888	2.2468	2.2043	2.1601	2.114
16	4.4940	3.6337	3.2388	3.0089	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.058
17	4.4513	3.5915	3.1968	2.9667	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.1040	2.0584	2.010
18	4.4139	3.5546	3.1599	2.9297	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0168	1.968
19	4.3807	3.5219	3.1274	2.8971	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.3080	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.930
20	4.3512	3.4928	3.0984	2.8681	2.7109	2.5990	2.5140	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.896
21	4.3248	3.4668	3.0725	2.8421	2.6848	2.5727	2.4876	2.4205	2.3660	2.3210	2.2504	2.1757	2.0960	2.0540	2.0102	1.9645	1.9165	1.865
22	4.3009	3.4434	3.0491	2.8187	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.9380	1.8894	1.838
23	4.2793	3.4221	3.0280	2.7975	2.6400	2.5277	2.4422	2.3748	2.3201	2.2747	2.2038	2.1282	2.0476	2.0050	1.9605	1.9139	1.8648	1.812
24	4.2597	3.4028	3.0088	2.7783	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9838	1.9390	1.8920	1.8424	1.789
25	4.2417	3.3852	2.9912	2.7587	2.6010	2.4883	2.4026	2.3351	2.2801	2.2345	2.1629	2.0869	2.0055	1.9623	1.9170	1.8695	1.8195	1.768
26	4.2252	3.3690	2.9752	2.7426	2.5848	2.4721	2.3863	2.3188	2.2635	2.2177	2.1459	2.0695	1.9878	1.9444	1.9000	1.8520	1.8015	1.748
27	4.2100	3.3541	2.9604	2.7278	2.5700	2.4573	2.3715	2.3040	2.2485	2.2025	2.1305	2.0538	1.9718	1.9281	1.8835	1.8350	1.7835	1.730
28	4.1960	3.3404	2.9467	2.7141	2.5563	2.4435	2.3577	2.2902	2.2345	2.1883	2.1161	2.0391	1.9568	1.9128	1.8680	1.8190	1.7670	1.713
29	4.1830	3.3277	2.9340	2.7014	2.5436	2.4308	2.3450	2.2775	2.2217	2.1755	2.1031	2.0258	1.9432	1.9000	1.8545	1.8050	1.7530	1.698
30	4.1709	3.3158	2.9223	2.6896	2.5318	2.4190	2.3332	2.2657	2.2099	2.1637	2.0913	2.0138	1.9310	1.8875	1.8415	1.7920	1.7395	1.683
40	4.0847	3.2317	2.8387	2.6060	2.4482	2.3354	2.2496	2.1821	2.1263	2.0801	2.0077	1.9300	1.8470	1.8035	1.7570	1.7070	1.6545	1.598
60	4.0012	3.1504	2.7581	2.5252	2.3674	2.2546	2.1688	2.1013	2.0455	2.0000	1.9275	1.8495	1.7665	1.7225	1.6760	1.6250	1.5715	1.515
120	3.9201	3.0718	2.6802	2.4472	2.2894	2.1766	2.0908	2.0233	1.9675	1.9220	1.8495	1.7710	1.6875	1.6430	1.5960	1.5440	1.4895	1.433



**Table A-3****The Chi-Square Table**Numbers in the table represent Chi-square values whose area to the right equals  $p$ .

$df/p$	0.10	0.05	0.025	0.01	0.005
1	2.71	3.84	5.02	6.64	7.88
2	4.61	5.99	7.38	9.21	10.60
3	6.25	7.82	9.35	11.35	12.84
4	7.78	9.49	11.14	13.28	14.86
5	9.24	11.07	12.83	15.09	16.75
6	10.65	12.59	14.45	16.81	18.55
7	12.02	14.07	16.01	18.48	20.28
8	13.36	15.51	17.54	20.09	21.96
9	14.68	16.92	19.02	21.67	23.59
10	15.99	18.31	20.48	23.21	25.19
11	17.28	19.68	21.92	24.73	26.76
12	18.55	21.03	23.34	26.22	28.30
13	19.81	22.36	24.74	27.69	29.819
14	21.06	23.69	26.12	29.14	31.32
15	22.31	25.00	27.49	30.58	32.80
16	23.54	26.30	28.85	32.00	34.27
17	24.77	27.59	30.19	33.41	35.72
18	25.99	28.87	31.53	34.81	37.16
19	27.20	30.14	32.85	36.19	38.58
20	28.41	31.41	34.17	37.57	40.00
21	29.62	32.67	35.48	38.93	41.40
22	30.81	33.92	36.78	40.29	42.80
23	32.01	35.17	38.08	41.64	44.18
24	33.20	36.42	39.36	42.98	45.56
25	34.38	37.65	40.65	44.31	46.93
26	35.56	38.89	41.92	45.64	48.29
27	36.74	40.11	43.20	46.96	49.65
28	37.92	41.34	44.46	48.28	50.99
29	39.09	42.56	45.72	49.59	52.34
30	40.26	43.77	46.98	50.89	53.67
40	51.81	55.76	59.34	63.69	66.77
50	63.17	67.51	71.42	76.15	79.49