UNIVERSITY OF SWAZILAND
MAIN EXAMINATION, DECEMBER 2014

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER:	**SIGNALS AND SYSTEMS I**
COURSE NUMBER:	**EE331**
TIME ALLOWED:	**THREE HOURS**

INSTRUCTIONS:

1. There are five questions in this paper. **Answer any FOUR questions.**
2. Each question carries 25 marks.
3. Marks for different sections are shown on the right hand margin.
4. Sheets containing useful tables are attached at the end.

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THIS PAPER HAS EIGHT (8) PAGES INCLUDING THIS PAGE

QUESTION 1

a) Define the following terms:

i) Walsh Functions [2]
ii) Invertible system [2]

b) Given a signal

$$f(t) = \begin{cases} 1, & 0 < t < 2 \\ 2, & 2 < t < 4 \end{cases}$$

where $f(t)$ is periodic with a period of $T = 4$.

i) Determine the general expression for the Fourier series coefficients, a_k. [12]

ii) Calculate the average power, P_{av} of $f(t)$. [5]

c) Determine the period and fundamental frequency of the following function: [4]

$$f(t) = 8 + 4e^{-j4t} + 4e^{j4t}$$

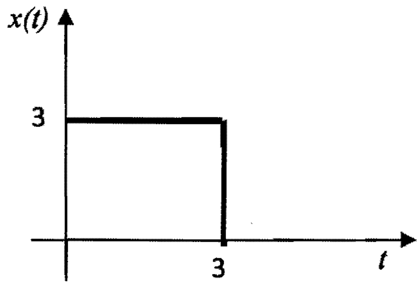
QUESTION 2

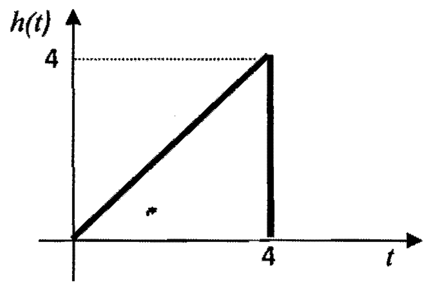
(a) A signal $x(t)$ is passed through a system with impulse response, $h(t)$ where

$$x(t) = \begin{cases} 3, & 0 < t < 3 \\ 0, & t \text{ otherwise} \end{cases}$$

$$h(t) = \begin{cases} t, & 0 < t < 4 \\ 0, & t \text{ otherwise} \end{cases}$$

as sketched below





(i) Find expressions for the output signal $y(t)$. The signal $y(t)$ may be divided into clearly defined time intervals. [10]

(ii) Find the maximum value of the response and the time at which it occurs. [3]

(b) Suppose a causal, linear, time-invariant, continuous-time system behaves according to the following differential equation:

$$y''(t) + 3y'(t) + 2y(t) = x'(t) + 3x(t)$$

where $x(t)$ and $y(t)$ are the input and output signals, respectively, to the system.

i) Find $H(s) = Y(s)/X(s)$ [2]

ii) Find the impulse response, $h(t)$, of the system. [4]

iii) Find the input, $x(t)$, to the system if the output is $y(t) = te^{-t}u(t)$. [6]

QUESTION 3

(a) The input-output signals of a system are given by $x(t)$ and $y(t)$ respectively. Draw a table such as given below and indicate with a "yes" or "no" whether the term at the head of each column is a correct description of the system given by the input-output equation on the left. [6]

System Equation	Linear	Time-invariant	Causal
$y(t) = \sqrt{x^2(t)}$			
$y[n] = 2\left(x[n+1]u[n] - x[n]\right) + 1$			

(b) Consider the pair of signals shown in Fig. Q3a:

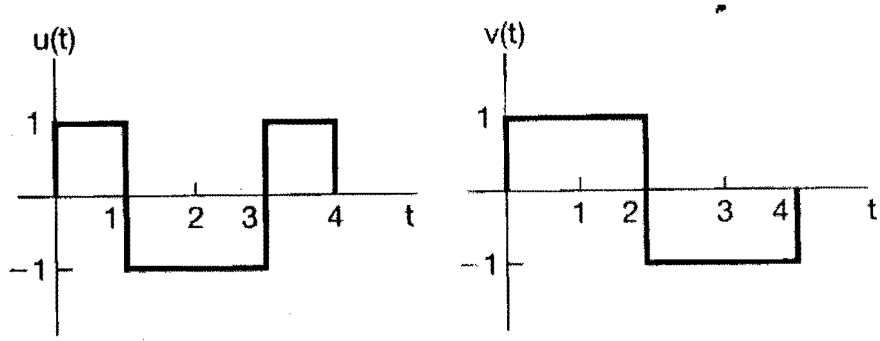


Fig. 3a

Determine whether the pair is orthogonal over the interval (0, 4). Justify your answer. [7]

(c) **(i)** Sketch the odd component of the signal shown in Fig. 4(b): [7]

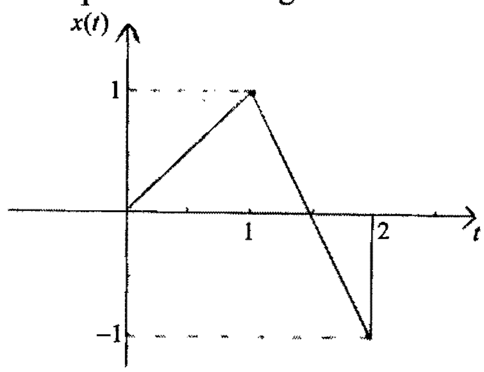


Fig. 4(b)

(ii) Is the signal $x(t)$ given above an energy or power signal? Justify your answer. [5]

QUESTION 4

(a) Express the unit rectangular pulse in terms of the unit step function. [2]

(b) State the three Dirichlet conditions. [3]

(c) Sketch the block diagram for a system having blocks with the following impulse responses:

$$\left[\left[h_1(t)+h_2(t)\right]*h_3(t)\right]+h_4(t)$$ [3]

(d) Given that $f(t)$ is the convolution of t and $\cos(2t)$ that is $f(t)=t*\cos(2t)$. Use the Laplace Transform Method to find the initial value of $f(t)$. [5]

(e) For the following signal, determine the location of the poles. [5]

$$x(t)=2\cos(3t)u(t)+3\sin(4t)u(t)$$

(f) Let $x(t)=\begin{cases} t+1, & -1\le t\le 0 \\ 1, & 0\le t\le 1 \\ 0, & otherwise \end{cases}$

i) Sketch the signal $x(t)$. [2]

ii) Sketch $y(t)=4x\left(\frac{t}{3}+2\right)$ [5]

QUESTION 5

(a) Find the Inverse Laplace Transform of

$$F(s) = \frac{5s - 12}{s^2 + 4s + 13}$$ [5]

(b) Using Laplace transforms solve the second-order linear differential equation

$$y''(t) + 4y'(t) + 4y(t) = 6e^{-2t}$$

with initial conditions $y(0) = -2$ and $y'(0) = 8$. [10]

(c) **(i)** Obtain the transfer function, $V_2(s)/V_1(s)$, of the circuit below, and [7]

(ii) Find its poles and zeros. [3]

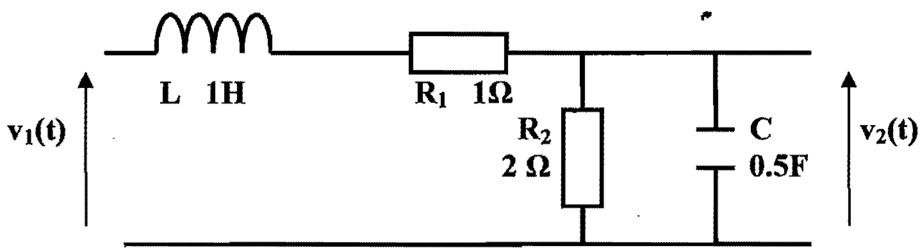


Table of Laplace Transforms

delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\|t\|}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$
frequency n-th differentiation	$t^nf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^nF^{(n)}(s)$
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

i) Time-shift (delay): $f(t-t_0) \xleftrightarrow{L} F(s)e^{-st_0}, \quad t_0 > 0$

ii) Time differentiation: $\frac{df(t)}{dt} \xleftrightarrow{L} sF(s) - f(0)$

iii) Time integration: $\int_0^t f(t)dt \xleftrightarrow{L} \frac{F(s)}{s}$

iv) Linearity: $af(t) + bg(t) \xleftrightarrow{L} aF(s) + bF(s)$

v) Convolution Integral: $x(t) * h(t) \xleftrightarrow{L} X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t} f(t) \xleftrightarrow{L} F(s-\alpha)$

vii) Multiplying by t: $tf(t) \xleftrightarrow{L} -\frac{dF(s)}{ds}$

viii) Scaling: $f(at) \xleftrightarrow{L} \frac{1}{a} F\left(\frac{s}{a}\right), \quad a > 0$

ix) Initial Value Theorem: $\lim_{s \to \infty} \{sF(s)\} = f(0)$

x) Final Value Theorem: $\lim_{s \to 0} \{sF(s)\} = f(\infty)$