University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Main Examination 2014

Title of Paper	:	**Control Engineering I**
Course Number	:	**EE431**
Time Allowed	:	**3 hrs**
Instructions	:	

1. **Answer any four (4) questions**
2. **Each question carries 25 marks**
3. **Useful information is attached at the end of the question paper**
4. **Special graph paper to be provided**

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The paper consists of nine (9) pages

Question 1

(a) Briefly compare and contrast open-loop control system versus closed loop control system. (4 Marks)

(b) Draw the block diagram of a closed loop control system for a disk drive. (4 Marks)

(c) Find the transfer function, $V_0(s)/V_i(s)$, for the circuit in figure 1(c) (6 Marks)

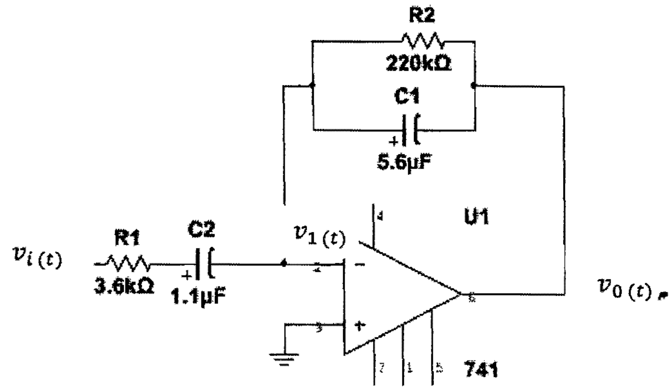


Figure 1 (c)

(d) Given the pole plot shown in figure 1(d), find $\xi, \omega_n, T_p, \%OS$ and T_s (5 Marks)

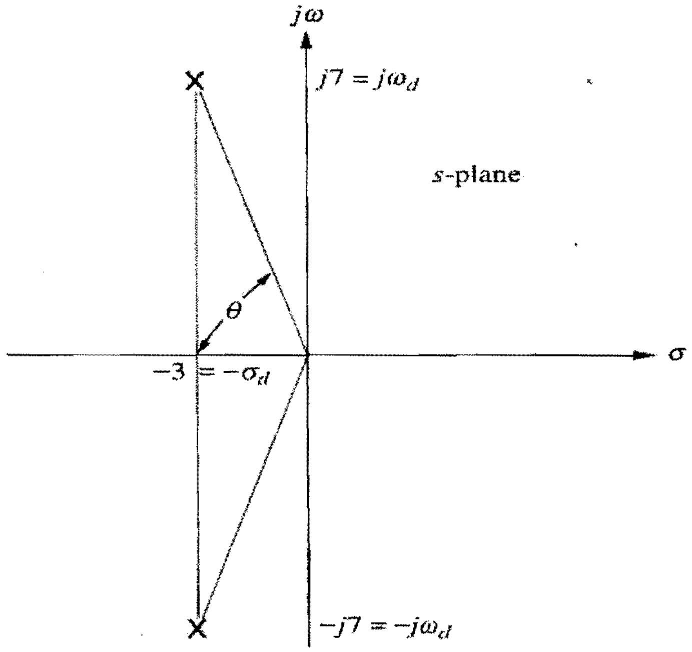


Figure 1 (d)

(e) Find the transfer function, $T(s) = Y(s)/R(s)$, for the following system represented in state space.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -5 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} r$$

$$y = [1 \quad 0 \quad 0]\mathbf{x}$$

Note: $\boldsymbol{T(s) = C(sI - A)^{-1}B + D}$

(6 Marks)

Question 2

(a) Based on the natural response definition of stability, explain for the the close-loop system case, the terms *stable, unstable* and *marginally stable.* (6 Marks)

(b) (i) Derive the transfer function of the non-inverting amplifier as shown in figure 2 (b). (7 Marks)

(ii) Then, calculate the transfer function given the following values for the circuit shown in figure 2(c).

(6 Marks)

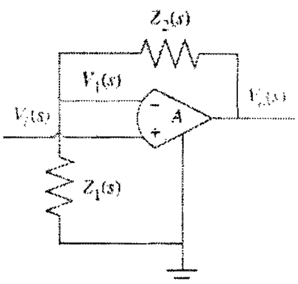


Figure 2(b)

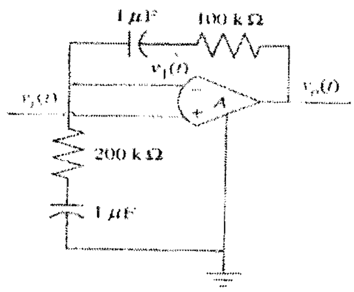


Figure 2 (c)

(c) Find the state-space representation in phase variable form for the system shown in figure 2(d). (6 Marks)

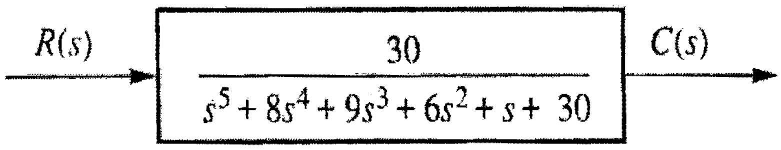


Figure 2 (d)

Question 3

(a) For the system shown in figure 3 (a), write the state equations and the output equation for the phase-variable representation. (8 Marks)

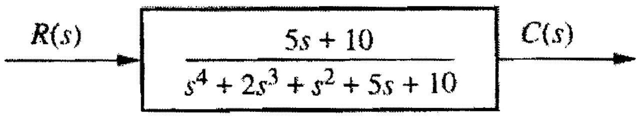


Figure 3 (a)

(b) Given the transfer function below, convert the transfer function into the following

$$T(s) = \frac{C(s)}{R(s)} = \frac{s^3 + s^2 + 7s + 1}{s^4 + 3s^3 + 5s^2 + 6s + 4}$$

(i) Phase-variable matrix form and draw the signal flow graph using the Phase-variable form. (4 Marks)

(ii) Show that the transfer function can be converted to a controller canonical matrix form and draw the signal flow graph using the controller canonical form. (4 Marks)

(c) Given the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$$

Find out how many poles are in the left half-plane, in the half-plane, and on the $j\omega$-axis. (9 Marks)

Question 4

(a) What information is contained in the specification $K_p = 100$? (4 Marks)

(b) Find the number of poles in the left half-plane, the right half-plane, and on the $j\omega$-axis for the system of figure 4 (b) (6 Marks)

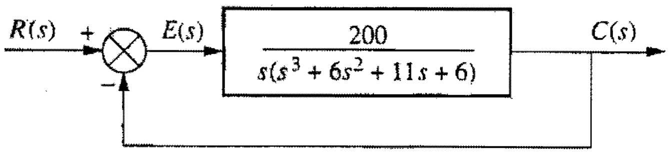


Figure 4 (b)

(c) Find the range of gain, K, for the system of figure 4 (c) that will cause the system to be *stable, unstable*, and *marginally stable*. Assume K > 0. (6 Marks)

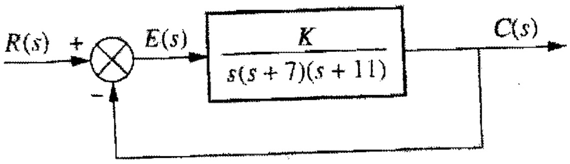


Figure 4(c)

(d) Using the Routh-Hurwitz criterion and the unity feedback system of figure 1(c) with

$$G(s) = \frac{K}{s(s+1)(s+2)(s+5)}$$

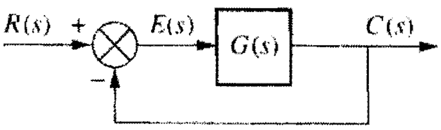


Figure 1(c)

(i) Find the range of K for stability [3]

(ii) Find the value of K for marginally stability [3]

(iii) Find the actual location of the closed-loop poles when the system is marginally stable. [3]

(9 Marks)

Question 5

(a) For the given transfer function, sketch the bode log magnitude diagram which shows how the log magnitude of the system is affected by changing input frequency. (7 Marks)

$$T(s) = \frac{1}{2s+100}$$

(b) Find the expression of frequency response for the system with a transfer function of the $G(s) = \frac{1}{1+2s}$, and then evaluate the magnitude and phase angles of frequency response at $\omega = 0.5\ rad/s$ and represent the result in a the complex plane.

(15 Marks)

(c) For the system in figure 5(c), evaluate the static error constants and find the expected error for the standard step, ramp, and parabolic inputs. (5 Marks)

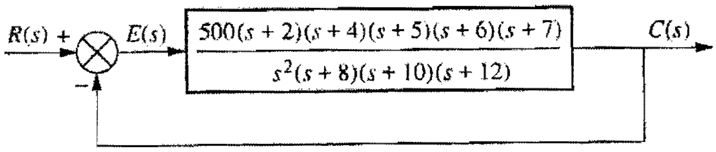


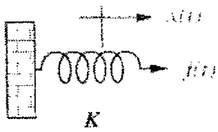
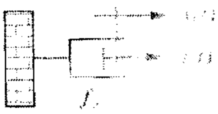
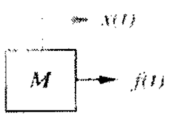
Figure 5 (c)

Table 1

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ = V (volts), $i(t)$ = A (amps), $q(t)$ = Q (coulombs), C = F (farads), R = Ω (ohms), G = ℧ (mhos), L = H (henries).

Table 2

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring  K	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
Viscous damper  f_v	$f(t) = f_v v(t)$	$f(t) = f_v\frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

Static Error Constants

For a step input, $u(t)$,

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

For a ramp input, $tu(t)$,

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)}$$

For a parabolic input, $\frac{1}{2}t^2u(t)$,

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s \to 0} s^2G(s)}$$

Position constant, K_p, where

$$K_p = \lim_{s \to 0} G(s)$$

Velocity constant, K_v, where

$$K_v = \lim_{s \to 0} sG(s)$$

Acceleration constant, K_a, where

$$K_a = \lim_{s \to 0} s^2G(s)$$

Table 3

Input	Steady-state error formula	Type 0 Static error constant	Type 0 Error	Type 1 Static error constant	Type 1 Error	Type 2 Static error constant	Type 2 Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$