# UNIVERSITY OF SWAZILAND 

## FACULTY OF SCIENCE <br> Department of Electrical and Electronic Engineering

# July 2016 <br> SUPPLEMENTARY EXAMINATION 

Title of the Paper:<br>Electromagnetic Fields I

Course Number: EE341
Time Allowed: Three Hours.

Instructions:

1. Answer all questions, no choice.
2. The answer is better neatly written in the space provided in the question book. Use the answer book as a scratch pad. Mark personal name and ID, and hand in all of them.
3. This paper has 7 pages, including this page.

Q1, 10 pts: Two point charges carry equal and opposite charge $+\mathrm{q} /-\mathrm{q}$, are located each at $d$ meters away from xy-plane on $z$-axis in the Cartesian coordinates. Find the zero potential surface.

Q2, 15 pts: A given scalar function is $(10-h(x, y))^{2}=\left(x^{2}+y^{2}\right)$, where $h(x, y)$ is the height of a cone, the peak of which is 10 shown in Fig. Q2-1, (i) calculate graphically the maximum change (gradient) of the height at the location near $P_{x}(4,4)$ and the direction of the change; (ii) calculate the same but analytically. Check if the two answers are close. (5 pts for (i) and 10 pts for (ii).)


Fig. Q2-1 of a cone.

Q3, 15 pts: Given the field pattern shown in Fig. Q3-1, (i) by inspection determine and mark the area which has curl $\neq 0$ or div $\neq 0$ or both $\neq 0$ of the pattern. Then (ii) analytically calculate the non-zero curl or divergence to prove. Take closed surface anywhere in the pattern but must be specified. The fields are in xy-plane only, no contribution in $z$-axis top and bottom. The closed surface may be bounded by a square or a circle. ( 5 pts for (i), 10 pts for (ii))
$\mathbf{A}=\hat{\mathbf{x}} x y^{2}+\hat{\mathbf{y}} x^{2} y$, for $-10 \leq x, y \leq 10$


Fig. Q3-1

Q4, $\mathbf{1 0} \mathbf{p t s}$ : Fill in the dual equation. (2 pts for each blank)

| Electric Fields | Magnetic Fields |
| :--- | :--- |
| $V=\frac{1}{4 \pi \varepsilon} \int_{v}^{q_{v}} \frac{q^{\prime}}{r}$ |  |
|  | $\oint_{c} \vec{H} \circ d \vec{l}=I$ |
| $\vec{\nabla} \times \vec{E}=0$ |  |
| $V_{c}=\frac{1}{C} \int_{0}^{t} i_{c} \cdot d t$ |  |
| Time constant <br> $=R \cdot C$ |  |

Q5, 10 pts: A magnetic circuit with all the pertinent dimensions in centimeter and cross sectional area $2 \times 10^{-3} \mathrm{Mtr}^{2}$ is shown in Fig. Q5-1. In the figure, the left and the right " C " are equal in size. Determine the current in the 1600 -turn coil to establish a flux density of 0.75 T in each air gap. Given the iron $\mathrm{u}_{\mathrm{r}}=1000$. (hint: using analogy of Ohm's law in magnetic
 circuit)

Q6, 10 pts: A long parallel plate cable has a width $w$ and a separation $d$ with insulation material $\varepsilon / \mu_{0}$. Consider no end fringing effects. (i) Find the total electric energy stored in the cable per meter, energized by a source charge $q_{1} \mathrm{Coul} / \mathrm{Mtr}$. (ii) Find the total magnetic energy stored in the cable per meter, energized by a total source current $\mathrm{I}_{\mathrm{s}}$. ( 5 pts for each)

Q7, 10 pts: Given a scalar function $f(x, y)=1$, find (i) $\int f \cdot d \vec{l}$ and (ii) $\int f \cdot d l$ along a triangle from $(-10,0)$ to $(0,10)$ to $(10,0)$ on top two quadrants in xy-plane, center at $(0,0) . \quad$ ( 5 pts for each (i) and (ii))

Q8, 10pts: A square coil of side $a$, shown in Fig. Q8-1 carries a current I. Determine the vector potential of this coil at the point on its axis $\overrightarrow{u_{z}}$ and z meters away from the coil plane.


Q9, 10 pts : In the air, there is a slab of the dielectric material with the constants $\varepsilon_{\mathrm{r}}$. Find the angle $\alpha_{4}$ in terms of $\alpha_{1}$. The geometry of the complex slab is shown in Fig. Q9-1. (hint:

$$
\left.\tan (a \pm b)=\frac{\tan (a) \pm \tan (b)}{1 \mp \tan (a) \cdot \tan (b)} \frac{\tan \theta_{2}}{\tan \theta_{1}}=\frac{\varepsilon_{2}}{\varepsilon_{1}}\right)
$$



