University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Supplementary Examination 2016

Title of Paper	:	**Control Engineering I**
Course Number	:	**EE431**
Time Allowed	:	**3 hrs**
Instructions	:	

1. **Answer any four (4) questions**
2. **Each question carries 25 marks**
3. **Useful information is attached at the end of the question paper**
4. **Special graph paper to be provided**

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR

The paper consists of eight (8) pages

Question 1

(a) For the system described by the transfer function, find the damping ratio (ξ), natural frequency (ω_n), percentage overshoot (%OS), peak time T_p and settling time T_s. [10]

$$T(s) = \frac{120}{S^2 + 12S + 120}$$

(b) Given the transfer function below, find the location of the poles and zeros, plot them on the s-plane, state the type of the response and then write an expression for the general form of the step response without solving for the inverse Laplace transform.[6]

$$T(s) = \frac{(S+5)}{(S+10)^2}$$

(c) Find the transfer function, X(s)/ F(s), for the system of figure 1 (c). [5]

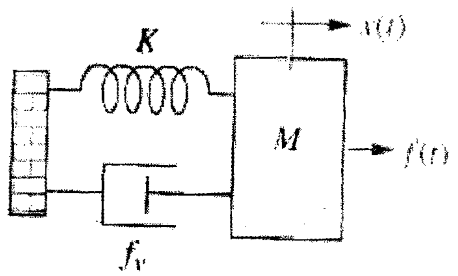


Figure 1 (c)

(d) State four (4) advantages of a closed-loop control system.[4]

Question 2

(a) Reduce the block diagram shown in figure 2 (a) to a single transfer function, $T(s) = C(s)/R(s)$. (Show all steps in full) [9]

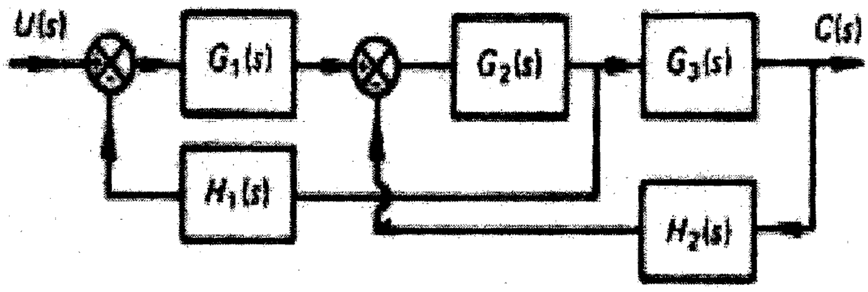


Figure 2 (a)

(b) Given the transfer function below, convert the transfer function into the following

$$T(s) = \frac{C(s)}{R(s)} = \frac{s^3 + s^2 + 7s + 1}{s^4 + 3s^3 + 5s^2 + 6s + 4}$$

(i) Phase-variable matrix form and draw the signal flow graph using the Phase-variable form. [4]

(ii) Show that the transfer function can be converted to a input feed-forward canonical matrix form and draw the signal flow graph using the input feed-forward canonical form. [4]

(c) Find the number of poles in the left half-plane, the right half-plane, and on the $j\omega - axis$ for the system of the figure 2 (c) below and state if the system is stable, unstable or imaginary stable based on the Routh table. [8]

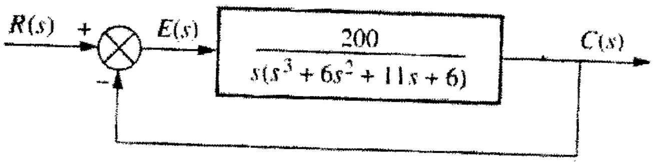


Figure 2 (c)

Question 3

(a) Find the transfer function relating the capacitor, $V_c(s)$, to the input voltage, V(s), in figure 3 (a) using the transform method. [5]

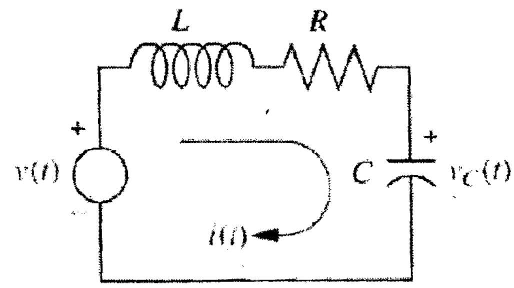


Figure 3 (a)

(b) Using Mason's rule, find the transfer function, T(s) = C(s)/R(s), for the system represented in figure 3(b) below.[10]

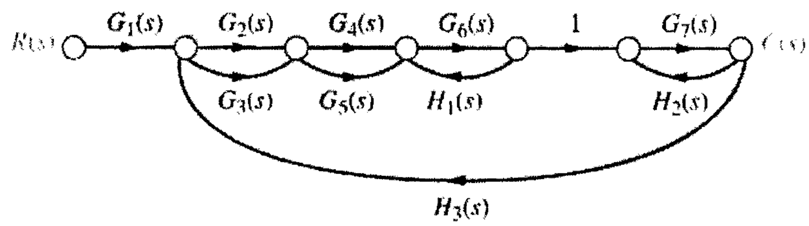


Figure 3(b)

(c) Based on the natural response definition of stability, explain based on close-loop relation when a system is said to be *stable, unstable* and *marginally stable.* [6]

(d) What information is contained in the specification $K_p = 1000$? [4]

Question 4

(a) Given the control system in figure 4(a), find the value of K so that there is 10% error in the steady state. [6]

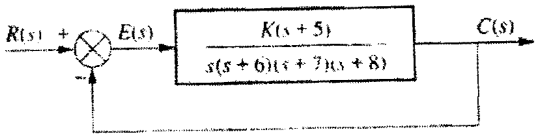


Figure 4(a)

(b) Determine the stability via reverse coefficients of the closed-loop transfer function. [8]

$$T(s) = \frac{10}{s^5+2s^4+3s^3+6s^2+5s+3}$$

(c) Given the following transfer function represent the systen in the parallel form. First represent the sum of individuals first-order systems and then draw the signal flow graph. Also, write the state equations and the vector-matrix form of the state equations. [10]

$$\frac{C(s)}{R(s)} = \frac{24}{(s+2)(s+3)(s+4)}$$

(d) Explain how a system is said to be stable based on BIBO definition.[1]

Question 5

(a) For the given transfer function, sketch the bode log magnitude diagram which shows how the log magnitude of the system is affected by changing input frequency.

(7 Marks)

$$T(s) = \frac{1}{2s+100}$$

(b) Find the expression of frequency response for the system with a transfer function of the $G(s) = \frac{1}{1+2s}$, and then evaluate the magnitude and phase angles of frequency response at $\omega = 0.5\, rad/s$ and represent the result in a the complex plane.

(13 Marks)

(c) For the system in figure 5(c), evaluate the static error constants and find the expected error for the standard step, ramp, and parabolic inputs. (5 Marks)

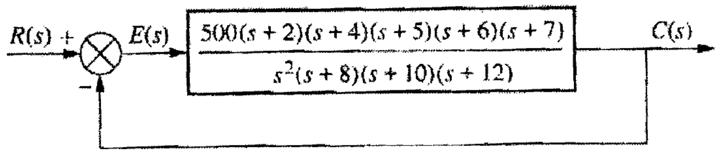


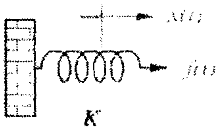
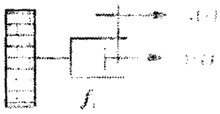
Figure 5 (c)

Table 1

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ = V (volts), $i(t)$ = A (amps), $q(t)$ = Q (coulombs), C = F (farads), R = Ω (ohms), G = ℧ (mhos), L = H (henries).

Table 2

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v\frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

Static Error Constants

For a step input, $u(t)$,

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

For a ramp input, $tu(t)$,

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)}$$

For a parabolic input, $\frac{1}{2}t^2u(t)$,

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s \to 0} s^2G(s)}$$

Position constant, K_p, where

$$K_p = \lim_{s \to 0} G(s)$$

Velocity constant, K_v, where

$$K_v = \lim_{s \to 0} sG(s)$$

Acceleration constant, K_a, where

$$K_a = \lim_{s \to 0} s^2G(s)$$

Table 3

		Type 0		Type 1		Type 2	
Input	**Steady-state error formula**	**Static error constant**	**Error**	**Static error constant**	**Error**	**Static error constant**	**Error**
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$