University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Main Examination 2015

Title of Paper	:	**Introduction to Digital Signal Processing**
Course Number	:	**EE443**
Time Allowed	:	**3 hrs**
Instructions	:	

1. **Answer any four (4) questions**
2. **Each question carries 25 marks**
3. **Useful information is attached at the end of the question paper**

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The paper consists of eight (8) pages

Question 1

(a) The impulse response of an electrical network is modelled by the exponentially decaying pulse

$$h(t) = 5\,x\;10^{3}e^{-\frac{t}{\tau}} \qquad for\ t \geq 0$$
$$= 0 \qquad for\ t < 0$$

The time-constant τ of the network, where $\tau = 1/\alpha$, is 1 ms. Work out the response of the network to an input sinewave of amplitude 3V and frequency 200 Hz. [5]

(b) Using the partial fraction expansion method, find the inverse of the following z-transform

$$Y(z) = \frac{z^2(z+1)}{(z-1)(z^2-z+0.5)}$$ [13]

(c) Explain how the information is stored in the compact-disc as one of the DSP real world application. Include illustrations where necessary. [5]

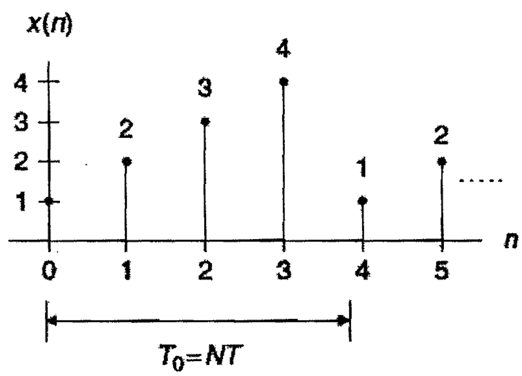
(d) State two (2) DSP applications. [2]

Question 2

(a) Find $x(n)$ if $X(z) = \frac{z^2}{(z-1)(z-0.5)^2}$ [12]

(b) How do we reduce the spectral leakage effects? [1]

(c) Consider the sequence



Assuming that $f_s = 100\ Hz$

(i) Evaluate its DFT X(k) [4]

(ii) Compute the amplitude and power spectrum. [8]

Question 3

(a) Design a second-order digital lowpass Butterworth filter with a cut- off frequency of 3.4 kHz at a sampling frequency of 8000 Hz. [15]

(b) Given the filter

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.5z^{-1}+0.25z^{-2}}$$

Realize H(z) and develop the difference equations using the following forms:

(i) Direct – Form I [2]

(ii) Direct- Form I [3]

(c) A second-order bandpass filter is required to satisfy the following specifications.

1. Sampling rate = 8000 Hz
2. 3 dB bandwidth: BW = 200 Hz
3. Narrow passband centered at f_0 = 1000 Hz
4. Zero gain at 0 Hz and 4000 Hz

Find the transfer function using the pole-zero placement method. [5]

Question 4

(a) Design a digital highpass Chebyshev filter with the following specifications: [20]

1. 0.5 dB ripple on passband at the frequency of 3000 Hz
2. 25 dB attenuation at the frequency of 1000 Hz
3. Sampling frequency of 8000 Hz

(b) Draw the digital signal processor based on the Harvard architecture and briefly describe the additional units. [5]

Question 5

(a) Given the fourth-order filter transfer function designed as

$$H(z) = \frac{0.5108z^2 + 1.0215z + 0.5108}{z^2 + 0.5654z + 0.4776} \times \frac{0.3730z^2 + 0.7460z + 0.3730}{z^2 + 0.4129z + 0.0790}$$

Realize the digital filter using the cascade (series) form via second-order sections using Direct-Form I and II. [10]

(b) Given an FIR filter transfer function

$$H(z) = 0.2 + 0.5z^{-1} - 0.3z^{-2} + 0.5z^{-3} + 0.2z^{-4}$$

Realize H (z) using each of the following specified methods:

(i) Transversal form, and write the difference equation for implementation. [3]
(ii) Linear phase form, and write the difference equation for implementation. [3]

(c) Consider a digital sequence sampled at the rate of 10 kHz. If we use a size of 1024 data points and apply the 1024-point DFT to compute the spectrum,

(i) Determine the frequency resolution. [1]
(ii) Determine the highest frequency in the spectrum. [2]

(d) Given the following difference equation with the input-output relationship of a certain initially relaxed DSP system (all initial conditions are zero),

$$y(n) - 0.4y(n-1) + 0.29y(n-2) = x(n) + 0.5\,x(n-1)$$

(i) Find the impulse response sequence $y(n)$ due to an impulse sequence $\delta(n)$ [6]

Table 1: Properties of z-transform

Property	Time Domain	z-Transform
Linearity	$ax_1(n)+bx_2(n)$	$aZ(x_1(n))+bZ(x_2(n))$
Shift theorem	$x(n-m)$	$z^{-m}X(z)$
Linear convolution	$x_1(n)*x_2(n)=\sum_{k=0}^{\infty} x_1(n-k)x_2(k)$	$X_1(z)X_2(z)$

Table 2: Partial fraction(s) and formulas for constant(s).

Partial fraction with the first-order real pole:

$$\frac{R}{z-p} \qquad R=(z-p)\frac{X(z)}{z}\bigg|_{z=p}$$

Partial fraction with the first-order complex poles:

$$\frac{Az}{(z-P)}+\frac{A^*z}{(z-P^*)} \qquad A=(z-P)\frac{X(z)}{z}\bigg|_{z=P}$$

P^* = complex conjugate of P

A^* = complex conjugate of A

Partial fraction with mth-order real poles:

$$\frac{R_m}{(z-p)}+\frac{R_{m-1}}{(z-p)^2}+\cdots+\frac{R_1}{(z-p)^m} \qquad R_k=\frac{1}{(k-1)!}\frac{d^{k-1}}{dz^{k-1}}\left((z-p)^m\frac{X(z)}{z}\right)\bigg|_{z=p}$$

Table 3: Chebyshev lowpass prototype transfer functions with 0.5 dB ripple ($\varepsilon = 0.3493$)

n	$H_P(s)$
1	$\frac{2.8628}{s+2.8628}$
2	$\frac{1.4314}{s^2+1.4256s+1.5162}$
3	$\frac{0.7157}{s^3+1.2529s^2+1.5349s+0.7157}$
4	$\frac{0.3579}{s^4+1.1974s^3+1.7169s^2+1.0255s+0.3791}$
5	$\frac{0.1789}{s^5+1.1725s^4+1.9374s^3+1.3096s^2+0.7525s+0.1789}$
6	$\frac{0.0895}{s^6+1.1592s^5+2.1718s^4+1.5898s^3+1.1719s^2+0.4324s+0.0948}$

Table 4: Summary of ideal impulse responses for standard FIR filters.

Filter Type	Ideal Impulse Response $h(n)$ (noncausal FIR coefficients)
Lowpass:	$h(n) = \begin{cases} \frac{\Omega_c}{\pi} & n = 0 \\ \frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Highpass:	$h(n) = \begin{cases} \frac{\pi-\Omega_c}{\pi} & n = 0 \\ -\frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Bandpass:	$h(n) = \begin{cases} \frac{\Omega_H-\Omega_L}{\pi} & n = 0 \\ \frac{\sin(\Omega_H n)}{n\pi} - \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Bandstop:	$h(n) = \begin{cases} \frac{\pi-\Omega_H+\Omega_L}{\pi} & n = 0 \\ -\frac{\sin(\Omega_H n)}{n\pi} + \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$

Causal FIR filter coefficients: shifting $h(n)$ to the right by M samples.
Transfer function:

$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_{2M} z^{-2M}$
where $b_n = h(n - M)$, $n = 0, 1, \cdots, 2M$

Table 5: 3 dB Butterworth lowpass prototype transfer functions ($\varepsilon = 1$)

n	$H_P(s)$
1	$\frac{1}{s+1}$
2	$\frac{1}{s^2+1.4142s+1}$
3	$\frac{1}{s^3+2s^2+2s+1}$
4	$\frac{1}{s^4+2.6131s^3+3.4142s^2+2.6131s+1}$
5	$\frac{1}{s^5+3.2361s^4+5.2361s^3+5.2361s^2+3.2361s+1}$
6	$\frac{1}{s^6+3.8637s^5+7.4641s^4+9.1416s^3+7.4641s^2+3.8637s+1}$

Table 6: Analog lowpass prototype transformations

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l\omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l\omega_h}$, $W = \omega_h - \omega_l$

Table 6: Conversion from analog filter specifications to lowpass prototype specifications.

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$\nu_p = 1$, $\nu_s = \omega_{as}/\omega_{ap}$
Highpass: ω_{ap}, ω_{as}	$\nu_p = 1$, $\nu_s = \omega_{ap}/\omega_{as}$
Bandpass: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_p = 1, \nu_s = \frac{\omega_{ash}-\omega_{asl}}{\omega_{aph}-\omega_{apl}}$
Bandstop: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_p = 1, \nu_s = \frac{\omega_{aph}-\omega_{apl}}{\omega_{ash}-\omega_{asl}}$

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge; ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband; ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband; ω_o, geometric center frequency.

The Z-transform

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n) z^{-n}$	
2	$\delta(n)$	1	$\lvert z \rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z \rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z \rvert > 1$
5	$n^2 u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z \rvert > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\lvert z \rvert > \lvert a \rvert$
7	$e^{-na} u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z \rvert > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z \rvert > \lvert a \rvert$
9	$\sin(an)u(n)$	$\frac{z \sin(a)}{z^2 - 2z\cos(a) + 1}$	$\lvert z \rvert > 1$
10	$\cos(an)u(n)$	$\frac{z[z - \cos(a)]}{z^2 - 2z\cos(a) + 1}$	$\lvert z \rvert > 1$
11	$a^n \sin(bn)u(n)$	$\frac{[a \sin(b)]z}{z^2 - [2a\cos(b)]z + a^2}$	$\lvert z \rvert > \lvert a \rvert$
12	$a^n \cos(bn)u(n)$	$\frac{z[z - a\cos(b)]}{z^2 - [2a\cos(b)]z + a^{-2}}$	$\lvert z \rvert > \lvert a \rvert$
13	$e^{-an} \sin(bn)u(n)$	$\frac{[e^{-a} \sin(b)]z}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z \rvert > e^{-a}$
14	$e^{-an} \cos(bn)u(n)$	$\frac{z[z - e^{-a}\cos(b)]}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z \rvert > e^{-a}$
15	$2\lvert A \rvert \lvert P \rvert^n \cos(n\theta + \phi)u(n)$ where P and A are complex constants defined by $P = \lvert P \rvert \angle \theta, A = \lvert A \rvert \angle \phi$	$\frac{Az}{z-P} + \frac{A^* z}{z-P^*}$	