UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2017

TITLE OF PAPER	:	**PROBABILITY AND STATISTICS**
COURSE CODE	:	**EE 301**
TIME ALLOWED	:	**3 HOURS**
INSTRUCTIONS	:	**ANSWER ANY FIVE QUESTIONS.**
REQUIREMENTS	:	**SCIENTIFIC CALCULATOR AND STATISTICAL TABLES.**

Question 1

a) Three couples attend a dinner. Each of the six people chooses randomly a seat at a round table. What is the probability that no couple sits together.

(10 Marks)

b) Consider the three events:

A = {Monday it will rain}, P(A) = 0.8
B = {Tuesday it will rain}, P(B) = 0.9
C = {Wednesday it will rain}, P(C) = 0.8

(i) Find the minimum interval covering $P(A \cup B \cup C)$, the probability that at least one of the three days will rain.

(ii) Find the minimum interval covering $P(A \cap B \cap C)$, the probability that in all the three days it will rain.

(iii) Are A, B and C mutually exclusive?

(iv) Assume A, B and C are mutually independent. Repeat parts (i) and (ii). Compare your answers.

(v) Suppose A, B and C are pairwise independent and that A, B are independent given C. Show that A, B and C are mutually independent.

(1+1+2+4+2 Marks)

Question 2

a) In the Lottery, there are 49 numbered balls, and six of these are selected at random. A seventh ball is also selected, but this is only relevant if you get exactly five numbers correct. The player selects six numbers before the draw is made, and after the draw, counts how many numbers are in common with those drawn. If the player has selected exactly three of the balls drawn, then the player wins E1000. The order the balls are drawn in is irrelevant. What is the probability of winning exactly E1000 on the National Lottery?

(10 Marks)

b) .A CD has 12 tracks on it, and these are to be played in random order. Suppose that you have time to listen to only 5 tracks before you go out.

(i) What the probability that the 5 played will be the first 5 on the cover case?

(ii) You arrange for your CD player to play 5 tracks at random. What is the probability that the 5 tracks played are your 5 favourite tracks (in any order)?

(5+5 Marks)

Question 3

One factory has four production lines to produce bicycles. Of the total production, line 1 produces 10%, line 2 produces 20%, line 3 produces 30% and line 4 produces 40%. The rates for defective products for these four production lines are 5%, 4%, 3%, and 2% respectively.

a) What is the probability, p, that a randomly chosen bicycle is defective?

(5 Marks)

b) If a bicycle is found defective, what is the probability that it comes from production line 4?

(5 Marks)

c) If an independent agency, like Consumers' Report buys 25 bicycles at random, what is the probability that none of them are defective?

(5 Marks)

d) For the 25 bicycles mentioned in the previous part of this question, what are the mean and standard deviation of the number of defective ones?

(5 Marks)

Question 4

a) X has *p.d.f.* $f(x) = 2\theta x\, EXP\{-\theta x^2\}$ for $x > 0$ and f(x) is zero elsewhere. Let $Y = X^2$. Note that this is a one to one transformation for the range of X for which the *p.d.f.* is non-zero. Use the standard transformation of variables result to obtain the *p.d.f.* for Y.

(6 Marks)

b) X has a distribution with p.d.f. $f(x) = \frac{\theta}{2}e^{-\theta|x|}$ for all $-\infty < x < \infty$ (where the parameter $\theta >$ 0). Show that the mgf of X is $M_X(t) = \left(1 = \frac{t^2}{\theta^2}\right)^{-1}$ for $|t| > \theta$. Obtain the mean μ and variance σ^2.

(14 Marks)

Question 5

a) Suppose that a binary message – either 0 or 1 – must be transmitted by wire from location A to location B. However, the data sent over the wire are subject to a channel noise disturbance, so to reduce the possibility of error, the value 2 is sent over the wire when the message is 1, and the value −2 is sent when the message is 0. If X, X = {−2,2}, is the value sent at location A, the value received at location B, denoted as R, is given by

$$R = X + N$$

where N is the channel noise disturbance, which is independent of X. When the message is received at location B, the receiver decodes it according to the following rule:

if $R \geq 0.5$, then conclude that message 1 was sent

if $R < 0.5$, then conclude that message 0 was sent

Assuming that the channel noise, N, is a unit normal random variable and that the message 0 or 1 is sent with equal probability, what is the probability that we conclude that the wrong message was sent? what is the probability of error for this communication channel.

(10 Marks)

b) The number of meteors found by a radar system in any 30-second interval under specified conditions averages 1.81. Assume the meteors appear randomly and independently.

(i) What is the probability that no meteors are found in a one-minute interval?

(5 Marks)

(ii) What is the probability of observing at least five but not more than eight meteors in two minutes of observation?

(5 Marks)

Question 6

A continuous random variable X has cumulative distribution function:

$$F_X(x) = \begin{cases} 0, if\ x \leq 0 \\ \sqrt{x}, if\ 0 < x \leq 1 \\ 1, if\ x > 1 \end{cases}$$

a) Find the probability density function of X.

(4 Marks)

b) Calculate the expectation and variance of X.

(12 Marks)

c) Calculate the lower quartile of X.

(4 Marks)

Question 7

A company manufacturing light bulbs is testing a new model. The company is going to test the hypothesis that the mean life time is 1000 hours vs. the alternative hypothesis that it is less than 1000 hours at the significance level = 0.02. Assume that the population distribution for life time is approximately normal. A sample of 16 light bulbs are found to have sample mean $\bar{x} = 987.5$ hours and sample variance $S^2 = 400$.

a) State the critical region and answer whether the null hypothesis H_0 is rejected.

(12 Marks)

b) Find a 90% confidence interval for the population variance σ^2.

(8 Marks)