

**UNIVERSITY OF SWAZILAND**  
**FACULTY OF SCIENCE & ENGINEERING**  
**DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING**  
**SIGNALS AND SYSTEMS I**  
**COURSE CODE - EE331**  
**MAIN EXAMINATION**  
**DECEMBER 2016**  
**DURATION OF THE EXAMINATION - 3 HOURS**

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**INSTRUCTIONS TO CANDIDATES**

1. There are **FIVE** questions in this paper. Answer any **FOUR** questions.
2. Each question carries 25 marks.
3. Show all your steps clearly in any calculations/work.
4. Start each new question on a fresh page.
5. Useful Fourier and Laplace Transform properties are attached.
6. Make sure that this exam contains 7 pages including this one.

**DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.**

**QUESTION ONE (25 marks)**

(a) (10 pts) Define the following terms.

- (i) Periodic signal
- (ii) Discrete-time signal
- (iii) Memoryless system
- (iv) Time-invariant system
- (v) Linear system

(b) (6 pts) Sketch and label carefully the following signals.

- (i)  $x(t) = u(t + 2) + u(t - 3)$
- (ii)  $x(t) = e^t(u(t - 1) - u(t - 3))$
- (iii)  $x(n) = 2u(-n + 2) - u(n - 2)$

(c) (9 pts) Draw a table such as given below and indicate with a 'yes' or 'no' whether the term at the head of each column is a correct description of the system given by the input-output equation on the left.

System equation	Causal	Linear	Time-invariant
$y(t) = x(-t)$			
$y(n) = x^2(n - 1)$			
$y(t) = tx(2t)$			

**QUESTION TWO (25 marks)**(a) A signal  $x(t)$  is passed through a system with impulse response  $h(t)$  where

$$x(t) = \begin{cases} 2, & 0 < t < 3 \\ 0, & \text{otherwise} \end{cases}, \quad h(t) = \begin{cases} t, & 0 < t < 4 \\ 0, & \text{otherwise} \end{cases}$$

- (i) (15 pts) Find expressions for the output signal  $y(t)$ . The signal  $y(t)$  may be divided into clearly defined time intervals.
- (ii) (3 pts) Roughly sketch  $y(t)$ .

(b) (7 pts) A causal digital processor with input response  $h(n)$  is fed with input sequence  $x(n)$ , as given below. Determine the output  $y(n)$ .

$$h(n) = [3 \ 4 \ 2 \ 3], \quad x(n) = 2\delta(n) + \delta(n - 1) + 3\delta(n - 2) + 5\delta(n - 4)$$

↑

**QUESTION THREE (25 marks)**

(a) (13 pts) Let  $x(t) = \begin{cases} t, & 0 < t < 1 \\ 2 - t, & 1 \leq t \leq 2 \end{cases}$  be a periodic signal with fundamental period

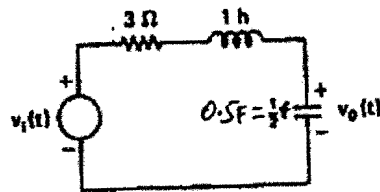
$T = 2$  and Fourier coefficients  $a_k$ .

(i) (3 pts) Determine the value of  $a_0$ .

(ii) (10 pts) Determine the Fourier series representation of  $\frac{dx(t)}{dt}$ .

(b) (12 pts)

(i) (5 pts) Determine the differential equation relating  $v_i(t)$  and  $v_o(t)$  for the RLC circuit below.



(ii) (7 pts) Suppose that  $v_i(t) = e^{-3t}u(t)$ , Determine  $v_o(t)$  for  $t > 0$

**QUESTION FOUR (25 marks)**

(a) (3+3+4 pts) Find the Laplace transform (LT) of the following continuous-time signals. You can assume unilateral LT.

(i) 5

(ii)  $5te^{-3t}$

(iii)  $5e^{-3t} \cos(5t)$

(b) (15 pts) Given the system defined by the following transfer function, find the output if the input is a unit ramp function  $x(t) = tu(t)$ .

$$H(s) = \frac{s + 1}{(s + 2)(s + 3)}$$

**QUESTION FIVE (25 marks)**

(5+6+7+7 pts) Let  $x(t)$  be a possibly complex valued time signal with Fourier transform  $X(\omega)$ . Express the Fourier transforms of the following time signals in terms of  $X(\omega)$ .

(i)  $x^*(t)$

(ii)  $\text{Im}\{x(t)\}$

(iii)  $x(t - 2) + x^*(-t + 2)$

(iv)  $e^{j4\pi t}x(3t + 3)$

**TABLE 1** Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$a > 0$
2	$e^{at}u(-t)$	$\frac{1}{a - j\omega}$	$a > 0$
3	$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	$a > 0$
4	$te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$a > 0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	
9	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	
10	$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$	
11	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
12	$\text{sgn } t$	$\frac{2}{j\omega}$	
13	$\cos \omega_0 t u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$	
14	$\sin \omega_0 t u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$	
15	$e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
16	$e^{-at} \cos \omega_0 t u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
17	$\text{rect}\left(\frac{t}{\tau}\right)$	$\tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi} \text{sinc}(Wt)$	$\text{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2} \text{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi} \text{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$	$\omega_0 = \frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling ( $a$ real)	$x(at)$	$\frac{1}{ a } X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting ( $\omega_0$ real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^t x(u) du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Table of Laplace Transforms

delta function	$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1
shifted delta function	$\delta(t-a)$	$\xleftrightarrow{\mathcal{L}}$	$e^{-as}$
unit step	$u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$
ramp	$tu(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2}{s^3}$
$n$ -th power	$t^n$	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s^{n+1}}$
exponential decay	$e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a t }$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2a}{a^2-s^2}$
	$te^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at} \sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at} \cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\xleftrightarrow{\mathcal{L}}$	$-F'(s)$
frequency $n$ -th differentiation	$t^n f(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^n F^{(n)}(s)$
time differentiation	$f'(t) = \frac{d}{dt} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s) - f(0)$
time 2nd differentiation	$f''(t) = \frac{d^2}{dt^2} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2 F(s) - sf(0) - f'(0)$
time $n$ -th differentiation	$f^{(n)}(t) = \frac{d^n}{dt^n} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau) d\tau = (u * f)(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s} F(s)$
frequency integration	$\frac{1}{t} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u) du$
time inverse	$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)^n}{s^n} + \frac{f^{-1}(0)}{s^{n-1}} + \frac{f^{-2}(0)}{s^{n-2}} + \dots + \frac{f^{-n}(0)}{s}$

## Properties of Laplace Transforms

- i) Time-shift (delay):  $f(t-t_0) \xrightarrow{L} F(s)e^{-st_0}$ ,  $t_0 > 0$
- ii) Time differentiation:  $\frac{df(t)}{dt} \xrightarrow{L} sF(s) - f(0)$
- iii) Time integration:  $\int_0^t f(t)dt \xrightarrow{L} \frac{F(s)}{s}$
- iv) Linearity:  $af(t) + bg(t) \xrightarrow{L} aF(s) + bF(s)$
- v) Convolution Integral:  $x(t) * h(t) \xrightarrow{L} X(s)H(s)$
- vi) Frequency-shift:  $e^{at} f(t) \xrightarrow{L} F(s-\alpha)$
- vii) Multiplying by  $t$ :  $tf(t) \xrightarrow{L} -\frac{dF(s)}{ds}$
- viii) Scaling:  $f(at) \xrightarrow{L} \frac{1}{a} F\left(\frac{s}{a}\right)$ ,  $a > 0$
- ix) Initial Value Theorem:  $\lim_{s \rightarrow \infty} \{sF(s)\} = f(0)$
- x) Final Value Theorem:  $\lim_{s \rightarrow 0} \{sF(s)\} = f(\infty)$