UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

SIGNALS AND SYSTEMS I

COURSE CODE - EE331

SUPPLEMENTARY EXAMINATION

JULY 2017

DURATION OF THE EXAMINATION - 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. There are **FIVE** questions in this paper. Answer any **FOUR** questions.

3. Each question carries 25 marks.

4. Show all your steps clearly in any calculations/work

5. Start each new question on a fresh page.

6. Useful Fourier and Laplace transform properties are **attached.**

7. Make sure that this exam contains 7 pages including this one.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION ONE (*25 marks*)

(a) (9 pts) Draw a table such as given below and indicate with a '√' or '×' whether the term at the head of each column is a correct description of the system given by the input-output equation on the left.

System equation	Causal	Linear	Time-invariant
$y(t) = x(-t)$			
$y(n) = x^2(n-1)$			
$y(t) = 2tx(2t)$			

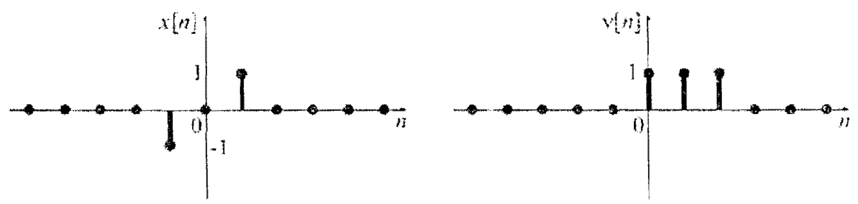
(b) (6 pts) Sketch and label carefully the following signals.

(i) $x(t) = u(t+3) + u(t-3)$

(ii) $x(t) = e^t(u(t-1) - u(t-3))$

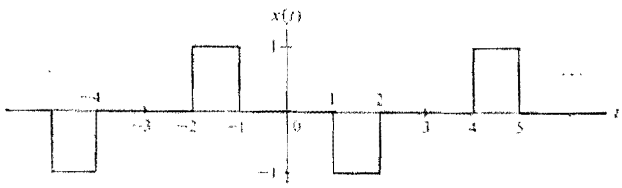
(iii) $x(n) = 2u(-n+2) - u(n-2)$

(c) (10 pts) Compute the convolution of the following two signals and plot the result.



QUESTION TWO (*25 marks*)

(a) (15 pts.) Determine the fundamental frequency, period and the coefficients of the Fourier series for the following signal x(t).



(b) (10 pts) Given a first-order system

$$y(n) + \frac{1}{2}y(n-1) = 2x(n)$$

Find the system response with initial condition of rest for $x(n) = \left(\frac{1}{2}\right)^n u(n)$.

Express your answer in closed form.

QUESTION THREE *(25 marks)*

Let $x_1(t) = 2(u(t) - u(t-1))$ and $x_2(t) = e^t(u(t) - u(t-2))$.

(i) Sketch $x_1(t)$ and $x_2(t)$.

(ii) Compute the convolution $y(t) = x_1(t) * x_2(t)$. You must do the computations either using the graphical or analytical method.

(iii) Sketch $y(t)$.

QUESTION FOUR *(25 marks)*

(a) (10 pts) Determine the Laplace transform (LT) and pole and zero locations for the following continuous-time signals.

(i) $e^{-3t}u(t)$

(ii) $5te^{-3t}$

(iii) $5e^{-3t}\cos(5t)\,u(t)$

(b) (15 pts) Determine $x(t)$ for each of the following Laplace transforms $X(s)$.

(i) $\frac{s+1}{s^2+5s+6}$, $Re\{s\} > -2$

(ii) $\frac{s^2-s+1}{s^2(s-1)}$, $0 < Re\{s\} < 1$

(iii) $\frac{s^2-s+1}{(s+1)^2}$, $-1 < Re\{s\}$

QUESTION FIVE *(25 marks)*

(a) (15 pts) Let $x(t)$ be a signal, and let $X(\omega)$ be its Fourier transform. Find the Fourier transform of the signal $y(t) = x\big(a(t-b)\big)$. Where $a \neq 0$ and $b \in \mathcal{R}$.

(b) (10 pts.) Using the definition of the Fourier transform, compute the Fourier transform of the following:

(i) $e^{at}u(-t)$, where $a > 0$.

(ii) $-u(t+1) + u(t)$

TABLE 1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t)+x_2(t)$	$X_1(\omega)+X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t-t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega-\omega_0)$
Time convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\,du$	$\frac{X(\omega)}{j\omega}+\pi X(0)\delta(\omega)$

Table of Laplace Transforms

delta function	$\delta(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	1
shifted delta function	$\delta(t-a)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}
unit step	$u(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$
ramp	$tu(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$
n-th power	t^n	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\|t\|}$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$
frequency n-th differentiation	$t^nf(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^nF^{(n)}(s)$
time differentiation	$f'(t)=\frac{d}{dt}f(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t)=\frac{d^2}{dt^2}f(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t)=\frac{d^n}{dt^n}f(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau=(u*f)(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\stackrel{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

i) Time-shift (delay): $f(t-t_0) \stackrel{L}{\longleftrightarrow} F(s)e^{-st_0}, \quad t_0 > 0$

ii) Time differentiation: $\frac{df(t)}{dt} \stackrel{L}{\longleftrightarrow} sF(s) - f(0)$

iii) Time integration: $\int_0^t f(t)dt \stackrel{L}{\longleftrightarrow} \frac{F(s)}{s}$

iv) Linearity: $af(t) + bg(t) \stackrel{L}{\longleftrightarrow} aF(s) + bF(s)$

v) Convolution Integral: $x(t) * h(t) \stackrel{L}{\longleftrightarrow} X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t} f(t) \stackrel{L}{\longleftrightarrow} F(s-\alpha)$

vii) Multiplying by t: $tf(t) \stackrel{L}{\longleftrightarrow} -\frac{dF(s)}{ds}$

viii) Scaling: $f(at) \stackrel{L}{\longleftrightarrow} \frac{1}{a} F\left(\frac{s}{a}\right), \quad a > 0$

ix) Initial Value Theorem: $\lim_{s \to \infty} \{sF(s)\} = f(0)$

x) Final Value Theorem: $\lim_{s \to 0} \{sF(s)\} = f(\infty)$