UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

SIGNALS AND SYSTEMS II

COURSE CODE - EE332

MAIN EXAMINATION

MAY 2017

DURATION OF THE EXAMINATION - 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. There are **FIVE** questions in this paper. Answer any **FOUR** questions.

3. Show all your steps clearly in any calculations/work.

4. State clearly any assumptions made.

5. Start each new question on a fresh page.

6. Useful Fourier transform and Z-transform tables are **attached**.

7. Make sure that this exam contains 9 pages including this one.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

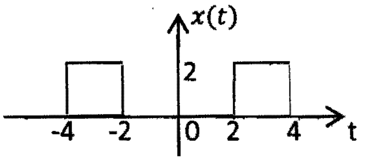
QUESTION ONE (*25 marks*)

(a) Suppose that $x(t)$ has the Fourier transform $u(2-\omega^2)$. Use the appropriate Fourier transform properties to find the Fourier transform of the signal $x(4-3t)$. [7]

(b) By using either calculation and/or Fourier transform table, find the Fourier transform $X(\omega)$ of the following signals. [8+10]

(i) $x(t) = e^{-3t}\cos(6t)\,u(t)$

(ii)



(Express your answer in terms of the sinc function.)

QUESTION TWO (*25 marks*)

(a) A particular LTI system is described by the difference equation

$$y(n) + \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = x(n) - x(n-1)$$

Determine the impulse response of the system. [6]

(b) Suppose that $h(n) = \left(\frac{1}{2}\right)^n u(n)$. Use discrete Fourier transforms to determine the response to the input signal $x(n) = \left(\frac{3}{4}\right)^n u(n)$. [6]

(c) Find the signal corresponding to the following Fourier transforms.

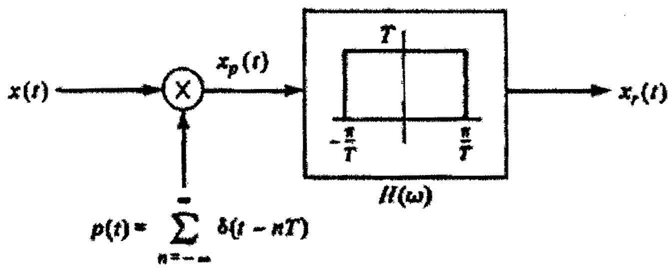
(i) $X(e^{j\omega}) = \frac{2}{1+\frac{1}{4}e^{-j(\omega-\frac{\pi}{2})}}$ [6]

(ii) $X(e^{j\omega}) = \frac{16e^{j2\omega}}{1-\frac{1}{4}e^{-j\omega}}$ [7]

QUESTION THREE (*25 marks*)

(a) State sampling theorem. Define Nyquist rate and Aliasing. [5]

(b) In the system shown in figure below, $x(t)$ is sampled with a periodic impulse train, and a reconstructed signal $x_r(t)$ is obtained from the samples by lowpass filtering.



The sampling period T is $1ms$, and $x(t)$ is a sinusoidal signal of the form $x(t) = \cos(2\pi f_0 t + \theta)$. For each of the following choices of f_0 and θ, determine $x_r(t)$. [20]

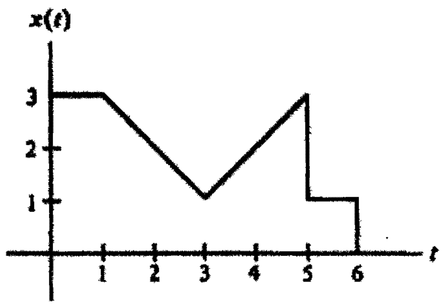
(i) $f_0 = 250Hz, \theta = \frac{\pi}{4}$

(ii) $f_0 = 750Hz, \theta = \frac{\pi}{2}$

(iii) $f_0 = 500Hz, \theta = \frac{\pi}{2}$

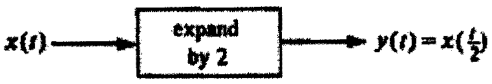
QUESTION FOUR *(25 marks)*

(a) What is amplitude modulation? Consider the signal $x(t)$ shown in figure below. [1+6+6]

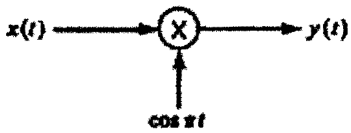


Draw y(t) for each of the following systems.

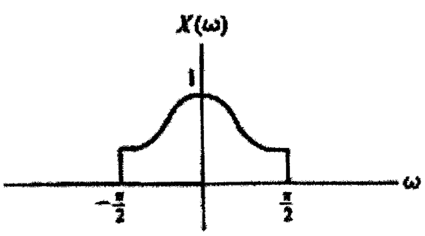
(i)



(ii)



(b) Suppose that $x(t)$ has the Fourier Transform shown in figure below. Find $Y(\omega)$ for each case in part (a). [6+6]



QUESTION FIVE (*25 marks*)

(a) Determine the z-transform (including the ROC) of the following sequences. Sketch the pole-zero plots and indicate the ROC on your sketch. [6]

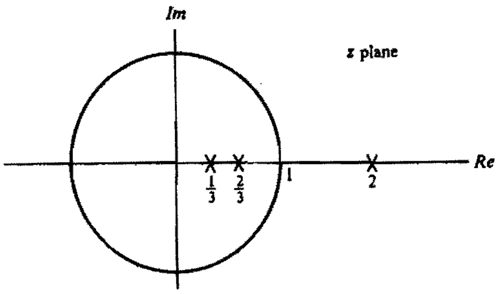
(i) $(\frac{2}{3})^n u(n)$

(ii) $\delta(n+2)$

(b) Find $x(n)$ from $X(z)$ below using partial fraction expansion, where $x(n)$ is known to be causal. [7]

$$X(z) = \frac{3+2z^{-1}}{2+3z^{-1}+z^{-2}}$$

(c) Shown in figure below is the pole-zero plot for the z-transform $X(z)$ of a sequence $x(n)$. [12]



Determine what can be inferred about the associated region of convergence from each of the following statement.

(i) $x(n)$ is right-sided.

(ii) $x(n)$ is left-sided.

(iii) The Fourier transform of $x(n)$ converges.

TABLE 1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\frac{2}{j\omega}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\frac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\operatorname{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\operatorname{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\|a\|} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\, du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

A. Properties of the discrete-time Fourier transform

Non-periodic signal	*Fourier transform*
$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$	$X(e^{j\omega}) \triangleq \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
$x[n]$, $y[n]$	$X(e^{j\omega})$, $Y(e^{j\omega})$ Periodic with period 2π
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
$x^*[n]$	$X^*(e^{j(-\omega)})$
$x[-n]$	$X(e^{j(-\omega)})$
$x_{(m)}[n] \coloneqq \begin{cases} x[n/m], & n \text{ multiple of } m \\ 0, & n \text{ not multiple av } m \end{cases}$	$X(e^{j(m\omega)})$
$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
$x[n] - x[n-1]$	$\left(1 - e^{j\omega}\right)X(e^{j\omega})$
$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-e^{j\omega}}X(e^{j\omega}) + \pi X(0)\sum_{k=-\infty}^{\infty}\delta(\omega - 2\pi k)$
$nx[n]$	$j\frac{d}{d\omega}X(e^{j\omega})$
If $x[n]$ is real valued then	
$x[n]$	$\begin{cases} X(e^{j\omega}) = X^*(e^{j(-\omega)}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{j(-\omega)})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{j(-\omega)})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{j(-\omega)})\rvert \\ \arg\{X(e^{j\omega})\} = -\arg\{X(e^{j(-\omega)})\} \end{cases}$
$x_e[n] = \mathcal{E}\{x[n]\}$	$\Re\{X(e^{j\omega})\}$
$x_o[n] = \mathcal{O}\{x[n]\}$	$j\Im\{X(e^{j\omega})\}$

Parsevals relation for non-periodic signals

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

B. Discrete-time Fourier transform table

$x[n]$	$X(e^{j\omega})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$\sum_{k=-\infty}^{\infty} \delta(n-kN)$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$
1	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$e^{j\omega_o n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - \omega_o - 2\pi k)$
$\cos\omega_o n$	$\pi \sum_{k=-\infty}^{\infty} [\delta(\omega-\omega_o-2\pi k) + \delta(\omega+\omega_o-2\pi k)]$
$\sin\omega_o n$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} [\delta(\omega-\omega_o-2\pi k) - \delta(\omega+\omega_o-2\pi k)]$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$a^n u(n), \quad \lvert a \rvert < 1$	$\frac{1}{1-ae^{-j\omega}}$
$(n+1)a^n u[n], \quad \lvert a \rvert < 1$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{(n+m-1)!}{n!(m-1)!} a^n u[n], \quad \lvert a \rvert < 1$	$\frac{1}{(1-ae^{-j\omega})^m}$
$\frac{1}{1-a^2} a^{\lvert n \rvert}, \quad \lvert a \rvert < 1$	$\frac{1}{1+a^2-2acos\omega}$
$\begin{cases} 1, & \lvert n \rvert \le N_1 \\ 0, & N_1 < \lvert n \rvert \le \frac{N}{2} \end{cases}$ period N	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega \frac{2\pi k}{N}\right)$
$\begin{cases} 1, & \lvert n \rvert \le N_1 \\ 0, & \lvert n \rvert > N_1 \end{cases}$	$\frac{\sin\omega\left(N_1+\frac{1}{2}\right)}{\sin\frac{\omega}{2}}$
$\begin{cases} \frac{\sin Wn}{\pi n} = \frac{W}{\pi}\mathrm{sinc}\frac{Wn}{\pi} \\ 0 < W < \pi \end{cases}$	$\begin{cases} 1, & \lvert\omega\rvert \le W \\ 0, & W < \lvert\omega\rvert \le \pi \end{cases}$ period 2π

Table of Z-Transforms

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n) z^{-n}$	
2	$\delta(n)$	1	$\lvert z\rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z\rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
7	$e^{-an}u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z\rvert > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
9	$\sin(an)u(n)$	$\frac{z\sin(a)}{z^2 - 2z\cos(a) + 1}$	$\lvert z\rvert > 1$
10	$\cos(an)u(n)$	$\frac{z[z-\cos(a)]}{z^2 - 2z\cos(a) + 1}$	$\lvert z\rvert > 1$
11	$a^n \sin(bn)u(n)$	$\frac{[a\sin(b)]z}{z^2 - [2a\cos(b)]z + a^2}$	$\lvert z\rvert > \lvert a\rvert$
12	$a^n \cos(bn)u(n)$	$\frac{z[z - a\cos(b)]}{z^2 - [2a\cos(b)]z + a^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
13	$e^{-an}\sin(bn)u(n)$	$\frac{[e^{-a}\sin(b)]z}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z\rvert > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\frac{z[z - e^{-a}\cos(b)]}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z\rvert > e^{-a}$

Properties of Z-Transforms

Linearity: $ax_1[k] + bx_2[k] \Leftrightarrow aX_1(z) + bX_2(z)$

Time Reversal: $x[-k] \Leftrightarrow X(1/z)$

Summation: $\sum_{n=-\infty}^{k} x[n] \Leftrightarrow \frac{zX(z)}{z-1}$

Initial Value: $x[0] = \lim_{z \to \infty} X(z)$

Final Value: $x[\infty] = \lim_{z \to 1}(z-1)X(z)$

Convolution: $x[k] * h[k] \Leftrightarrow X(z)H(z)$

Differencing: $x[k] - x[k-1] \Leftrightarrow \left(1 - z^{-1}\right)X(z)$

Differentiation: $-kx[k] \Leftrightarrow z\frac{d}{dz}X(z)$

Time Shifting: $x[n-n_o] \Leftrightarrow z^{-n_o}X(z), n_o \geq 0$

$$x[n+n_o] \Leftrightarrow z^{n_o}\left(X(z) - \sum_{m=0}^{n_o-1} x[m]z^{-m}\right), n_o \geq 0$$