University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Main Examination 2016

Title of Paper	**:**	**Control Engineering I**
Course Number	**:**	**EE431**
Time Allowed	**:**	**3 hrs**
Instructions	**:**	

1. **Answer any four (4) questions**
2. **Each question carries 25 marks**
3. **Useful information is attached at the end of the question paper**

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The paper consists of eight (8) pages including this page

Question 1

(a) The root locus of a feedback system with open-loop poles at -1, -3 and open zeros at -8, -10 is shown in Figure 1 below. Study the diagram and then answer the following questions.

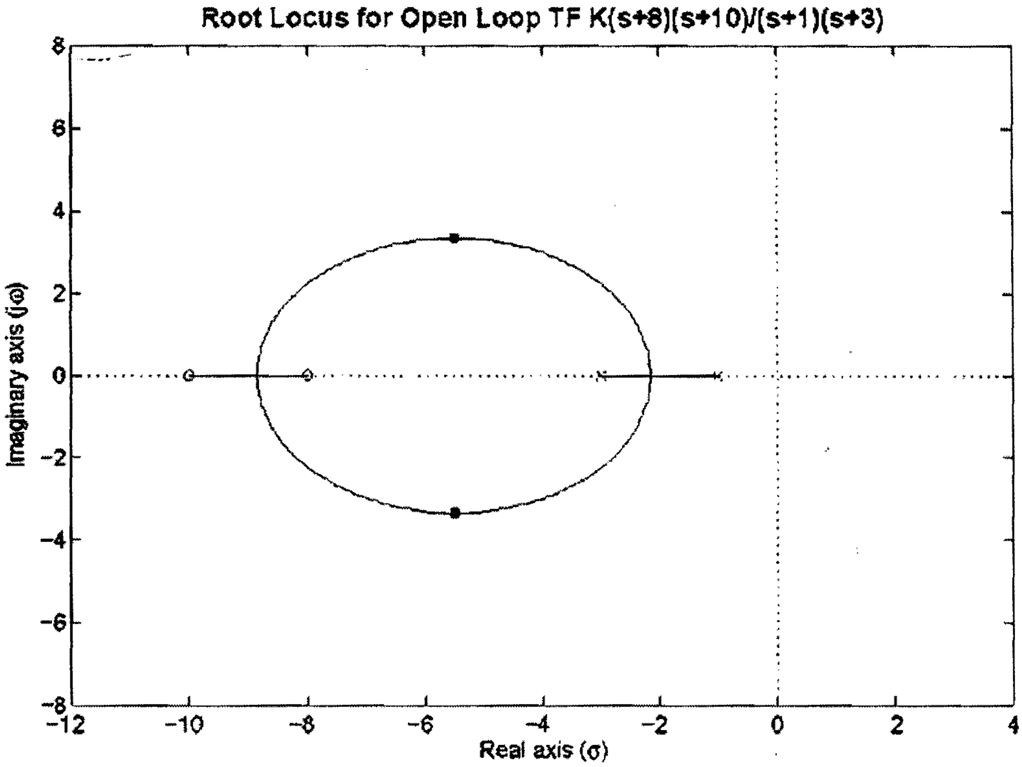


Figure 1

(i) Can the response of the system be tuned (approximately) to a settling time of 1 sec with an appropriate choice of feedback gain K. Justify your answer graphically. If your answer is "yes", there is no need to compute the value of K that would give this settling time. [5]

(ii) Can the response of this system be tuned (approximately) to an overshoot of 4.32 % with an appropriate choice of feedback gain K? Justify your answer graphically. If your answer is "yes", there is no need to compute the value of K that would give this amount of overshoot. <u>Hint:</u> A damping ratio of $\zeta = 1/\sqrt{2}$ would approximately yield the desired overshoot value. [5]

(b) A ship autopilot is designed to maintain a vessel on a set heading while being subjected to a series of disturbances such as wind, waves and current as shown in the Figure 2. The actual heading is measured by a gyro-compass (or magnetic compass in a smaller vessel), and compared with the desired heading, dialled into the autopilot by the ship's master. The autopilot, or controller, computes the demanded rudder angle and sends a control signal to the steering gear. The actual rudder angle is monitored by a rudder angle sensor and compared with the demanded rudder angle to form a control loop. The rudder provides a control moment on the hull to drive the actual heading towards the desired heading while the wind, waves and current produce moments that may help or hinder this action. Develop the block diagram of the system explained above. [10]

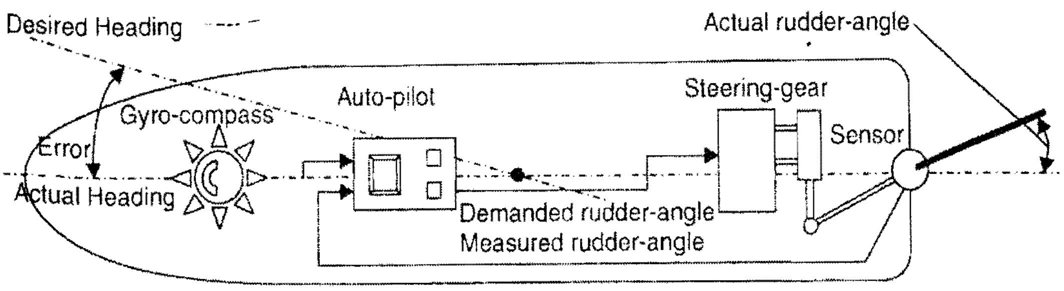


Figure 2: Ship Autopilot control system

(c) Given the function below, find the sampled time function.

$$F(z) = \frac{0.5\,z}{(z-0.5)(z-0.7)} \qquad [5]$$

Question 2

(a) Reduce the system shown in figure 3 (a) to a single transfer function using the block reduction method. [10]

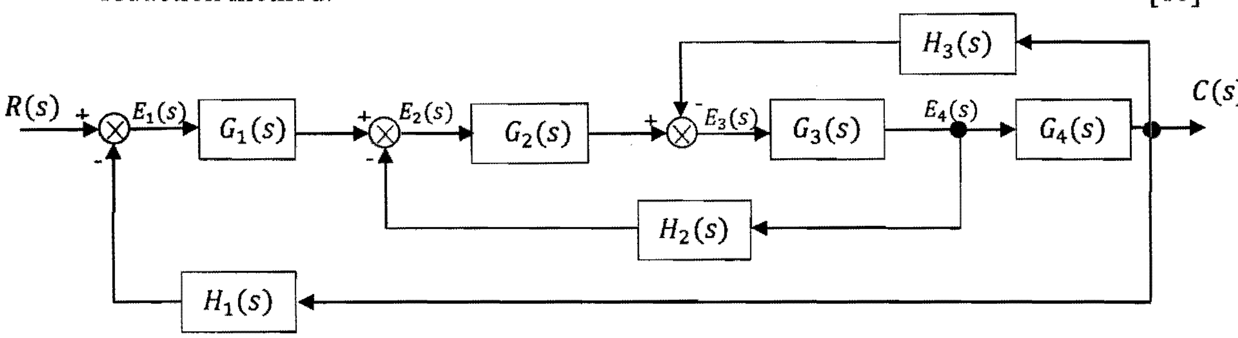


Figure 3a

(b) We are given a feedback system whose open-loop transfer function is $\frac{K(s+20)}{(S+2)(S+4)(S+10)}$

where K is the feedback gain. Evaluate the system's close-loop behaviour using the root locus technique.

(i) How many asymptotes are there in this system's root locus? What are the asymptotes angles? [2]

(ii) Where is the asymptotes real-axis intercept? [2]

(iii) Sketch the root locus based on the information from the previous questions. There is no need to annotate break-in/away points or imaginary axis intercepts, if any. [6]

(iv) If you had to recommend this system to a customer, what would you advise with respect to increasing the feedback gain K indefinitely? [3]

(c) Given the figure below (Figure 3b), represent the system in state space in phase-variable form. [2]

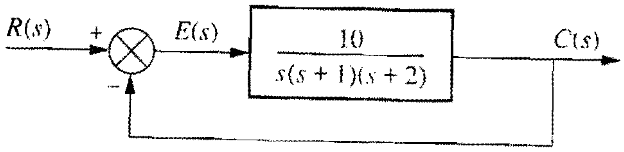
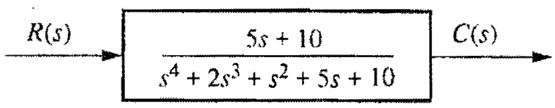


Figure 3b

Question 3

(a) For the system shown below, write the state equations and the output equation for the phase variable-representation. In addition, draw the equivalent block diagram. [15]



(b) For the transfer function,

$$T(s) = \frac{20}{s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20}$$

Find out how many poles are in the right half-plane, the left half-plane, and on the $j\omega$ – axis. Is the system stable? [10]

Question 4

(a) You are given the transfer function of an antenna position control system. Find the range of the preamplifier gain required to keep the closed-loop system stable. [5]

$$T(s) = \frac{6.63\,K}{s^3+101.71s^2+171s+6.63K}$$

(b) Given a unity feedback system with

$$G(s) = \frac{4}{s(s^6+2s^5+2s^4-4s^3-s^2+2s-2)}$$

Find out how many poles of the closed-loop transfer function lie in the right half-plane, the left half-plane, and on the $j\omega - axis$. [10]

(c) Find the transfer function, $G(s) = C(s)/R(s)$, for each of the following systems represented in state space. [10]

$$\dot{x} = \begin{bmatrix} 2 & 3 & -5 \\ 0 & 5 & 3 \\ -3 & -2 & -4 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} r$$

$$y = [1 \ \ 4 \ \ 7]\ \mathbf{x}$$

Question 5

(a) Find the breakaway and break-in points for the root locus in **Figure 4a** using differential calculus. [5]

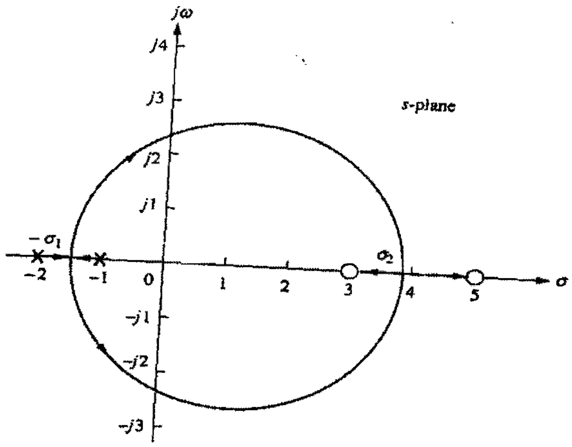


Figure 4 a

(b) Given the unity feedback system of **Figure 4b**, find the angle of departure from the complex poles and sketch the root locus. [5]

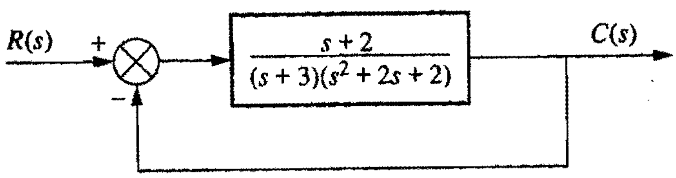


Figure 4 b

(c) Given the figure below (Figure 4 c),

(i) Find the transfer function for the operational amplifier circuit [5]

(ii) If $R_1 = R_2$, $R_3 C = 1/10$: find the step response of the filter. [10]

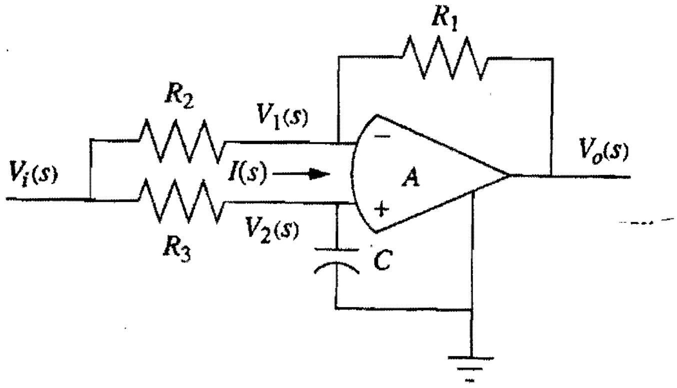


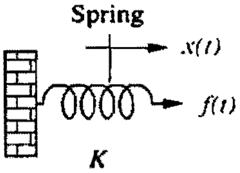
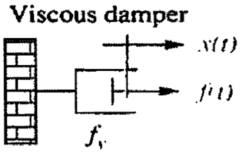
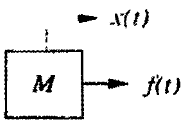
Figure 4 c

Table 1

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ = V (volts), $i(t)$ = A (amps), $q(t)$ = Q (coulombs), C = F (farads), R = Ω (ohms), G = ℧ (mhos), L = H (henries).

Table 2

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v\frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds²/meter).

Table 3

Input	Steady-state error formula	Type 0 Static error constant	Type 0 Error	Type 1 Static error constant	Type 1 Error	Type 2 Static error constant	Type 2 Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

Static Error Constants

For a step input, $u(t)$,

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s\to 0} G(s)}$$

For a ramp input, $tu(t)$,

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s\to 0} sG(s)}$$

For a parabolic input, $\frac{1}{2}t^2u(t)$,

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s\to 0} s^2G(s)}$$

Position constant, K_p, where

$$K_p = \lim_{s\to 0} G(s)$$

Velocity constant, K_v, where

$$K_v = \lim_{s\to 0} sG(s)$$

Acceleration constant, K_a, where

$$K_a = \lim_{s\to 0} s^2G(s)$$