University of Swaziland
Faculty of Science
Department of Electrical and Electronic Engineering
Main Examination 2016

Title of Paper : **Introduction to Digital Signal Processing**

Course Number : **EE443**

Time Allowed : **3 hrs**

Instructions :

1. **Answer any four (4) questions**
2. **Each question carries 25 marks**
3. **Useful information is attached at the end of the question paper**

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The paper consists of eight (7) pages

Question 1

(a) Considering the sequence $x(0) = 1, x(1) = 2, x(2) = 3, x(3) = 4$, and given f_s = 100 Hz, T=0.01 seconds, compute the amplitude spectrum, phase spectrum and power spectrum using the Hamming Window function. [20]

(b) Find the inverse z-transform of the following function [5]:

$$X(z) = \frac{5z}{z^2 - z + 1}$$

Question 2

(a) Given the filter

$$H(z) = \frac{1 - 0.9z^{-1} - 0.1z^{-2}}{1 + 0.3z^{-1} - 0.04z^{-2}},$$

Realize $H(z)$ and develop the difference equation using the following form

(i) Cascade (series) form via first order-sections [5]
(ii) Parallel form via first order-sections [5]

(b) Given a sequence $x(n)$ for $0 \leq n \leq 3$, where $x(0) = 1, x(1) = 2, x(2) = 3, x(3) = 4$. Evaluate its DFT $X(k)$ using the decimation-in-time FFT method. [5]

(c) Find $x(n)$ if $X(z) = \frac{z^2}{(z-1)(z-0.5)^2}$ [10]

Question 3

(a) Convert the following transfer function into its difference equation. [3]

$$H(z) = \frac{z^2 - 0.5z + 0.36}{z^2}$$

(b) Design a bandpass FIR filter with following specifications: [10]

Lower stopband = 0-500 Hz
Passband = 1600- 2300 Hz
Upper stopband = 3500 – 4000 Hz
Stopband attenuation = 50 dB
Passband ripple = 0.05 dB

Sampling rate = 8000 Hz

State all the coefficients.

(c) Describe the basic mechanism of circular buffering for a buffer having eight data samples. [5]
(d) What is Gibb's effect in FIR digital filters? How does it originate and how to remedy this problem? [4]
(e) State the advantage of the floating point processor. [3]

Question 4

Design a digital bandpass Chebyshev filter with the following specifications:

- Center frequency of 2.5 kHz
- Passband bandwidth of 200 Hz, 0.5 dB ripple on passband
- Lower stop frequency of 1.5 kHz, upper stop frequency of 3.5 kHz
- Stopband attenuation of 10 dB
- Sampling frequency of 8000 Hz

Show all your work! [25]

Question 5

Design a digital bandstop Butterworth filter with the following specifications:

- Center frequency of 2.5 kHz
- Passband width of 200 Hz and ripple of 3dB.
- Stopband width of 50Hz and attenuation of 10 dB
- Sampling frequency of 8000 Hz

Show all your work! [25]

Table 1: Properties of z-transform

Property	Time Domain	z-Transform
Linearity	$ax_1(n) + bx_2(n)$	$aZ(x_1(n)) + bZ(x_2(n))$
Shift theorem	$x(n-m)$	$z^{-m}X(z)$
Linear convolution	$x_1(n)*x_2(n) = \sum_{k=0}^{\infty} x_1(n-k)x_2(k)$	$X_1(z)X_2(z)$

Table 2: Partial fraction(s) and formulas for constant(s).

Partial fraction with the first-order real pole:	
$\frac{R}{z-p}$	$R = (z-p)\frac{X(z)}{z}\Big\rvert_{z=p}$
Partial fraction with the first-order complex poles:	
$\frac{Az}{(z-P)} + \frac{A^*z}{(z-P^*)}$	$A = (z-P)\frac{X(z)}{z}\Big\rvert_{z=P}$
P^* = complex conjugate of P	
A^* = complex conjugate of A	
Partial fraction with mth-order real poles:	
$\frac{R_m}{(z-p)} + \frac{R_{m-1}}{(z-p)^2} + \cdots + \frac{R_1}{(z-p)^m}$	$R_k = \frac{1}{(k-1)!}\frac{d^{k-1}}{dz^{k-1}}\left((z-p)^m \frac{X(z)}{z}\right)\Big\rvert_{z=p}$

Table 3: 3 dB Butterworth lowpass prototype transfer functions ($\varepsilon = 1$)

n	$H_P(s)$
1	$\frac{1}{s+1}$
2	$\frac{1}{s^2+1.4142s+1}$
3	$\frac{1}{s^3+2s^2+2s+1}$
4	$\frac{1}{s^4+2.6131s^3+3.4142s^2+2.6131s+1}$
5	$\frac{1}{s^5+3.2361s^4+5.2361s^3+5.2361s^2+3.2361s+1}$
6	$\frac{1}{s^6+3.8637s^5+7.4641s^4+9.1416s^3+7.4641s^2+3.8637s+1}$

Table 4: Summary of ideal impulse responses for standard FIR filters.

Filter Type	Ideal Impulse Response **$h(n)$ (noncausal FIR coefficients)**
Lowpass:	$h(n) = \begin{cases} \frac{\Omega_c}{\pi} & n = 0 \\ \frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Highpass:	$h(n) = \begin{cases} \frac{\pi - \Omega_c}{\pi} & n = 0 \\ -\frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Bandpass:	$h(n) = \begin{cases} \frac{\Omega_H - \Omega_L}{\pi} & n = 0 \\ \frac{\sin(\Omega_H n)}{n\pi} - \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Bandstop:	$h(n) = \begin{cases} \frac{\pi - \Omega_H + \Omega_L}{\pi} & n = 0 \\ -\frac{\sin(\Omega_H n)}{n\pi} + \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$

Causal FIR filter coefficients: shifting $h(n)$ to the right by M samples.
Transfer function:

$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_{2M} z^{-2M}$
where $b_n = h(n - M)$, $n = 0, 1, \cdots, 2M$

Table 5: Chebyshev lowpass prototype transfer functions with 0.5 dB ripple (ε = 0.3493)

n	$H_P(s)$
1	$\frac{2.8628}{s+2.8628}$
2	$\frac{1.4314}{s^2+1.4256s+1.5162}$
3	$\frac{0.7157}{s^3+1.2529s^2+1.5349s+0.7157}$
4	$\frac{0.3579}{s^4+1.1974s^3+1.7169s^2+1.0255s+0.3791}$
5	$\frac{0.1789}{s^5+1.1725s^4+1.9374s^3+1.3096s^2+0.7525s+0.1789}$
6	$\frac{0.0895}{s^6+1.1592s^5+2.1718s^4+1.5898s^3+1.1719s^2+0.4324s+0.0948}$

Table 6: Analog lowpass prototype transformations

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l\omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l\omega_h}$, $W = \omega_h - \omega_l$

Table 6: Conversion from analog filter specifications to lowpass prototype specifications.

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$\nu_p = 1$, $\nu_s = \omega_{as}/\omega_{ap}$
Highpass: ω_{ap}, ω_{as}	$\nu_p = 1$, $\nu_s = \omega_{ap}/\omega_{as}$
Bandpass: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_p = 1, \nu_s = \frac{\omega_{ash}-\omega_{asl}}{\omega_{aph}-\omega_{apl}}$
Bandstop: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$\nu_p = 1, \nu_s = \frac{\omega_{aph}-\omega_{apl}}{\omega_{ash}-\omega_{asl}}$

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge; ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband; ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband; ω_o, geometric center frequency.

The Z-transform

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$\lvert z\rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z\rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
7	$e^{-na}u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z\rvert > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
9	$\sin(an)u(n)$	$\frac{z\sin(a)}{z^2-2z\cos(a)+1}$	$\lvert z\rvert > 1$
10	$\cos(an)u(n)$	$\frac{z[z-\cos(a)]}{z^2-2z\cos(a)+1}$	$\lvert z\rvert > 1$
11	$a^n \sin(bn)u(n)$	$\frac{[a\sin(b)]z}{z^2-[2a\cos(b)]z+a^2}$	$\lvert z\rvert > \lvert a\rvert$
12	$a^n \cos(bn)u(n)$	$\frac{z[z-a\cos(b)]}{z^2-[2a\cos(b)]z+a^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
13	$e^{-an}\sin(bn)u(n)$	$\frac{[e^{-a}\sin(b)]z}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z\rvert > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\frac{z[z-e^{-a}\cos(b)]}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z\rvert > e^{-a}$
15	$2\lvert A\rvert \lvert P\rvert^n \cos(n\theta+\phi)u(n)$ where P and A are complex constants defined by $P = \lvert P\rvert\angle\theta, A = \lvert A\rvert\angle\phi$	$\frac{Az}{z-P}+\frac{A^*z}{z-P^*}$	