University of Swaziland
Faculty of Science and Engineering
Department of Electrical and Electronic Engineering

Main Examination 2018

Title of paper: **Introduction to Digital Signal Processing**

Course Number: **EE443**

Time allowed: 3 hours

Instructions:

1. Answer any FOUR (4) questions
2. Each question carries 25 marks
3. Marks for each question are shown at the right hand margin
4. Useful information is attached at the end of the examination paper

This paper contains 6 pages including this one.

This paper should not be opened until permission has been granted by the invigilator

Question 1

(a) Show that the accumulator system defined by the equation below is time invariant [6]

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

(b) Given the sequences $x_1[n] = 3\delta[n] - 2\delta[n-1]$ and $x_2[n] = 2\delta[n] - \delta[n-1]$, find the convolution sum given by the equation below, using the z-transforms $x_{conv}[n] = x_1[n] \star x_2[n]$; where $\star$ denotes convolution [5]

(c) Using the partial fraction expansion method, find the inverse of the following z-transform

$$Y(z) = \frac{1}{(1-z^{-1})(1-0.5z^{-1})}$$ [7]

(d) Realize the following digital filter using a direct form II:

$$H(z) = \frac{0.7157 + 1.4314z^{-1} + 0.7151z^{-2}}{1 + 1.3490z^{-1} + 0.5140z^{-2}}$$

[7]

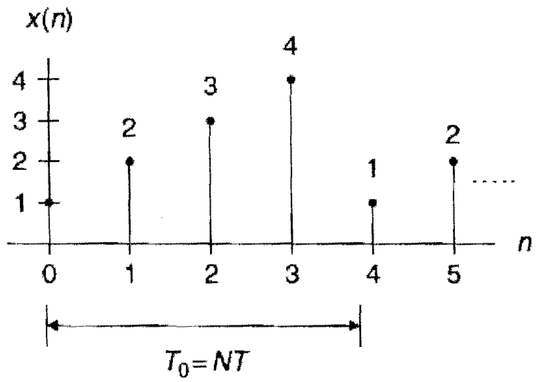
Question 2

(a) Find $x(n)$ if

i) $X[z] = \frac{z^{-4}}{z-1} + z^{-6} + \frac{z^{-3}}{z+0.5}$ [4]

ii) $X[z] = 2 + \frac{4z}{z-1} + z^{-6} + \frac{z}{z-0.5}$ [4]

(b) Consider the sequence



Assuming that $f_s = 100\ Hz$

i) Evaluate its DFT $X(k)$ [4]

ii) Compute the amplitude, phase, and power spectrum. [12]

iii) Compute the frequency resolution [1]

Question 3

(a) Given the normalized lowpass filter with a cutoff frequency of 1rad/sec below:

$$H_p(s) = \frac{1}{s+1}$$

Use $H_p(s)$ above and the BLT to design a corresponding digital IIR lowpass filter with a cutoff frequency of 15 kHz at a sampling frequency of 90 kHz [15]

(b) Considering the sequence $x[0] = 4, x[1] = 2,\ x[2] = 3$, and given $f_s = 100Hz$, $T = 0.01s$ compute the magnitude spectrum of $X[k]$ where $X[k]$ is the DFT of $x_w[n]$, using the using the Hamming window given by $w_{hm}(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)$. [8]

(c) Describe how to control spectral leakage. [2]

Question 4

(a) Given the FIR filter:

$$y(n) = 0.1x(n) + 0.25x(n-1) + 0.2x(n-2)$$

Determine the transfer function, the filter length, the non-zero coefficients and the impulse response. [9]

(b) Show that the equation below

$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{nk}\ for\ k = 1,2,\dots,N-1.$$

Can be simplified to the following two terms

$$X[2m] = \sum_{n=0}^{\frac{N}{2}-1} a[n]W_{N/2}^{mn}\ for\ k = 1,2,\dots,\frac{N}{2}-1$$

$$X[2m+1] = \sum_{n=0}^{\frac{N}{2}-1} b[n]W_N^n W_{N/2}^{mn}\ for\ k = 1,2,\dots,\frac{N}{2}-1$$

[8]

(c) Draw the eight-point FFT. [8]

Question 5

(a) Given the fourth-order filter transfer function designed as

$$H(z) = \frac{0.5108z^2 + 1.0215z + 0.5108}{z^2 + 0.5654z + 0.4776} \times \frac{0.3730z^2 + 0.7460z + 0.3730}{z^2 + 0.4129z + 0.0790}$$

Realize the digital filter using the cascade (series) form via second-order sections using Direct-Form I and Direct-Form II. [13]

(b) Write the mathematical definitions of the following sequences

i) Unit sample sequence [2]
ii) Exponential sequence [2]
iii) Unit step sequence [2]

(c) Write the synthesis and analysis equations of the DFT [6]

Table 1: Properties of z-transform

Property	Time Domain	z-Transform
Linearity	$ax_1(n) + bx_2(n)$	$aZ(x_1(n)) + bZ(x_2(n))$
Shift theorem	$x(n-m)$	$z^{-m}X(z)$
Linear convolution	$x_1(n)*x_2(n) = \sum_{k=0}^{\infty} x_1(n-k)x_2(k)$	$X_1(z)X_2(z)$

Table 2: Partial fraction(s) and formulas for constant(s).

Partial fraction with the first-order real pole:	
$\frac{R}{z-p}$	$R = (z-p)\frac{X(z)}{z}\Big\rvert_{z=p}$
Partial fraction with the first-order complex poles:	
$\frac{Az}{(z-P)} + \frac{A^*z}{(z-P^*)}$	$A = (z-P)\frac{X(z)}{z}\Big\rvert_{z=P}$
P^* = complex conjugate of P	
A^* = complex conjugate of A	
Partial fraction with mth-order real poles:	
$\frac{R_m}{(z-p)} + \frac{R_{m-1}}{(z-p)^2} + \cdots + \frac{R_1}{(z-p)^m}$	$R_k = \frac{1}{(k-1)!}\frac{d^{k-1}}{dz^{k-1}}\left((z-p)^m \frac{X(z)}{z}\right)\Big\rvert_{z=p}$

Table 3: Analog lowpass prototype transformations

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$

The Z-transform

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n) z^{-n}$	
2	$\delta(n)$	1	$\lvert z \rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z \rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z \rvert > 1$
5	$n^2 u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z \rvert > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\lvert z \rvert > \lvert a \rvert$
7	$e^{-na} u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z \rvert > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z \rvert > \lvert a \rvert$
9	$\sin(an)u(n)$	$\frac{z \sin(a)}{z^2 - 2z\cos(a) + 1}$	$\lvert z \rvert > 1$
10	$\cos(an)u(n)$	$\frac{z[z - \cos(a)]}{z^2 - 2z\cos(a) + 1}$	$\lvert z \rvert > 1$
11	$a^n \sin(bn)u(n)$	$\frac{[a \sin(b)]z}{z^2 - [2a\cos(b)]z + a^2}$	$\lvert z \rvert > \lvert a \rvert$
12	$a^n \cos(bn)u(n)$	$\frac{z[z - a\cos(b)]}{z^2 - [2a\cos(b)]z + a^{-2}}$	$\lvert z \rvert > \lvert a \rvert$
13	$e^{-an} \sin(bn)u(n)$	$\frac{[e^{-a} \sin(b)]z}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z \rvert > e^{-a}$
14	$e^{-an} \cos(bn)u(n)$	$\frac{z[z - e^{-a}\cos(b)]}{z^2 - [2e^{-a}\cos(b)]z + e^{-2a}}$	$\lvert z \rvert > e^{-a}$
15	$2\lvert A \rvert \lvert P \rvert^n \cos(n\theta + \phi)u(n)$ where P and A are complex constants defined by $P = \lvert P \rvert \angle \theta, A = \lvert A \rvert \angle \phi$	$\frac{Az}{z-P} + \frac{A^* z}{z-P^*}$	