# University of Swaziland <br> Faculty of Science and Engineering Department of Electrical and Electronic Engineering 

Main Examination 2018

Title of paper: Digital Communication Systems
Course Number: EE543

## Time allowed: $\mathbf{3}$ hours

## Instructions:

1. Answer any FOUR (4) questions
2. Each question carries 25 marks
3. Marks for each question are shown at the right hand margin
4. Useful information is attached at the end of the paper.

This paper contains 5 pages including this one.

This paper should not be opened until permission has been granted by the invigilator.

## Question 1

(a) Assuming a 4-bit ADC channel accepts analogue input ranging from 0 to 5 volts, determine the following:
(i) Number of quantization levels
(ii) Step size of the quantizer or resolution
(iii) Quantization level when the analogue voltage is 3.2 volts
(iv) Binary code produced
(v) Quantization error
(b) Consider the binary sequence 1001011 . Draw the waveforms for the following signalling formats.
(i) Split phase (Manchester) signalling format
(ii) Bipolar RZ signalling format
(iii) AMI (alternate mark inversion) RZ signalling format
(c) A communication channel of bandwidth 75 kHz is required to transmit binary data at a rate of $0.1 \mathrm{Mb} / \mathrm{s}$ using raised-cosine pulses. Determine the roll-off factor $\alpha$.
(d) List and describe four channels of digital communications
(e) State four advantages of digital communications

## Question 2

(a) Describe the sampling process and state the conditions for successful recovery of the signal from the samples.
(b) Consider an AWGN channel with 4 kHz bandwidth and the noise power spectral density $\frac{\eta}{2}=10^{-12} \mathrm{~W} / \mathrm{Hz}$. The signal power required at the receiver is 0.1 mW . Calculate the capacity of this channel.
(c) Given the following random binary sequence 10011011001110 .
(i) Perform partial response signalling, that is, show the signal formed assuming the initialization bit is 1 .
(ii) Draw a transmitted Delta Modulation waveform
(d) A DMS X has four symbols $x_{1}, x_{2}, x_{3}, x_{4}$ with probabilities $P\left(x_{1}\right)=0.4, P\left(x_{2}\right)=$ $0.3, P\left(x_{3}\right)=0.2, P\left(x_{4}\right)=0.1$
(i) Calculate $\mathrm{H}(\mathrm{X})$
(ii) Find the amount of information contained in the messages $x_{1} x_{2} x_{1} x_{3}$ and $x_{4} x_{3} x_{3} x_{2}$ and compare with the $\mathrm{H}(\mathrm{X})$ obtained in part (i).

## Question 3

(a) A DMS X has four symbols $x_{1}, x_{2}, x_{3}$ and $x_{4}$ with $P\left(x_{1}\right)=\frac{1}{2}, P\left(x_{2}\right)=\frac{1}{4}$, and $P\left(x_{3}\right)=$ $P\left(x_{4}\right)=\frac{1}{8}$. Construct a Shannon-Fano code for X , Show that this code has the optimum property that $n_{i}=I\left(x_{i}\right)$ and that the code efficiency is 100 percent.
(b) Describe how you can reduce a MPSK M-correlator receiver to an $\mathrm{N}=2$ correlator receiver
(c) Simplify the following QPSK equation into its orthogonal basis function and message points
$s_{i}(t)=\sqrt{\frac{2 E}{T}} \cos \left[(2 i-1) \frac{\pi}{4}\right] \cos \left(2 \pi f_{c} t\right)-\sqrt{\frac{2 E}{T}} \sin \left[(2 i-1) \frac{\pi}{4}\right] \sin \left(2 \pi f_{c} t\right), i=1, \ldots, 4$
(d) Describe the following errors as they are encountered in delta modulation:
(i) Granular noise

- (ii) Slope overload


## Question 4

(a) Show that the error probability of equal-energy signal for a BPSK receiver

$$
P_{B}=\int_{\gamma_{0}=\frac{\left(a_{1}-a_{2}\right)}{2}}^{\infty} \frac{1}{\sqrt{2 \pi} \sigma_{0}} \exp \left(-\frac{1}{2}\left(\frac{z-a_{1}}{\sigma_{0}}\right)^{2}\right) d z
$$

.. results in $Q\left(\sqrt{\frac{2 E_{b}}{N_{0}}}\right)$ in AWGN
(b) Find the BER for a BPSK system with a bit rate of $1 \mathrm{Mbits} / \mathrm{s}$ the received waveforms $s_{1}(t)=A \cos \left(\omega_{0} t\right)$ and $s_{2}(t)=-A \cos \left(\omega_{0} t\right)$ are coherently detected with a matched filter. The value of $A=10 \mathrm{mV}$. Assume that $N_{0}=10^{-11} \mathrm{~W} / \mathrm{Hz}$ and the signal power and energy is normalized relative to a $1 \Omega$ resistor
(c) Draw the structure of a digital communication system and describe the function of each block.
(d) Show that the code $C=\{0000,0101,1010,1111\}$ is a linear cyclic code. [6]

## Question 5

(a) A 100 kHz signal is sampled at the Nyquist rate and an 8 -level quantizer is used to quantize a signal.
(i) Calculate the number of bits to encode one sample
(ii) Calculate the number of bits per second transmitted .
(b) Given the generator sequences of a convolutional encoder $g_{1}=111$ and $g_{2}=101$,
(i) Draw the convolutional encoder
(ii) Find the impulse response of the encoder
(c) Show that the matched filter of signal $s(t)$ is given by

$$
h(t)=\left\{\begin{array}{lr}
s(t-T), & \text { for } 0 \leq t \leq T  \tag{5}\\
0, & \text { otherise }
\end{array}\right.
$$

(d) Consider OFDM
(i) State the property of the subcarriers that make OFDM symbols not to interfere with each other and write the equation for the subcarrier

- (ii) Draw the OFDM symbol and state the reason for using a cyclic prefix
(iii) Given that the OFDM symbol is $10 \mu s$ long with a cyclic prefix of $2 \mu s$, calculate the SNR loss due to the insertion of the cyclic prefix

Table 1

| $=$ | $Q(2)$ | $*$ | $Q(z)$ | $\%$ | O(1) | \% | $Q(-1$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 mo | 0.5000 | 1.00 | 0.1587 | 200 | 0.0228 | 300 | 0.00135 |
| 0.05 | 0.4801 | 105 | 0.1469 | 205 | 0.0202 | 3.05 | 0.00114 |
| 0.10 | 0.4602 | 1.10 | 0.1357 | 2.10 | 0.0179 | + 110 | 0.00097 |
| 0.15 | 0.4414 | 1.15 | 0.1251 | 2.15 | 0.0158 | 3.15 | 0.00082 |
| 0.30 | 0.4207 | 1.20 | 0.1151 | 220 | 0.0139 | 3.20 | 0.00069 |
| 0.23 | 0.4013 | 1.25 | (1) 1050 | 2.25 | 0.0122 | 3.35 | 0.00058 |
| 0.30 | 0.3621 | 1.31 | 0.0968 | 2., 11 | 0.0107 | 3. 30 | 0.00048 |
| 0.35 | 0.3632 | 1.35 | 00885 | 2.15 | 000094 | 3.35 | 0.00040 |
| 0.40 | 0.3446 | 1.40 | 0.0808 | 240 | 00082 | 3.40 | 0.00004 |
| 0.45 | 0.326 | 1.45 | 0.0635 | 245 | 0.007 | 3.45 | 0.00002 x |
| U. 50 | 0.3085 | 1.50 | 0.0668 | 250 | 0.0062 | 3.50 | 0,0mers |
| 0.55 | 0.2912 | 1.55 | 00006 | 2.55 | 0.0054 | 1.55 | 000019 |
| 0.60 | 0.2743 | 1.60 | 60548 | 260 | 0.0047 | 3.60 | 0.00016 |
| 065 | 0.2578 | 1.65 | 0.0495 | 265 | 0.6040 | 3.65 | 0,00013? |
| 0.70 | 0.2420 | 1.70 | 0.04 .46 | 2.70 | 06035 | 3.70 | 0.00011 |
| 0.75 | 0.2266 | 1.75 | 0.0401 | 275 | 0.0030 | 3.75 | 0.00009 |
| 0 NO | 0.2169 | 1.80 | 00155 | 2.80 | 0.9026 | 1.80 | 0.00007 |
| 085 | 0.19? | 1.85 | 06122 | 2.85 | 0.0122 | $3 \times 5$ | 0.00006 |
| 649 | 0.1841 | 1.90 | (1) 428? | 3.90 | 40019 | . 90 | 0.0000s |
| 0.95 | 11.111 | 1.45 | 1103.86 | 2.95 | 0.0016 | 3.95 | 0.000004 |
| 400 | 0.00003 |  |  |  |  |  |  |
| 4.25 | $10^{\circ}$ |  |  |  |  |  |  |
| 4.75 | $10^{6}$ |  |  |  |  |  |  |
| 5.20 | $10^{*}$ |  |  |  |  |  |  |
| 5.60 | $10^{*}$ |  |  |  |  |  |  |

