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UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

SIGNALS AND SYSTEMS I

COURSE CODE – EEE331/EE331

MAIN EXAMINATION

DECEMBER 2017

DURATION OF THE EXAMINATION - 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. There are **FIVE** questions in this paper. Answer any **FOUR** questions.

3. Each question carries 25 marks.

4. Show all your steps clearly in any calculations/work

5. Start each new question on a fresh page.

6. Useful Fourier and Laplace transform properties are **attached**.

7. Make sure that this exam contains 8 pages including this one.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION ONE *(25 marks)*

(a) (i) (3 pts) Sketch the odd component of the signal shown in Fig. 1. Show your work!

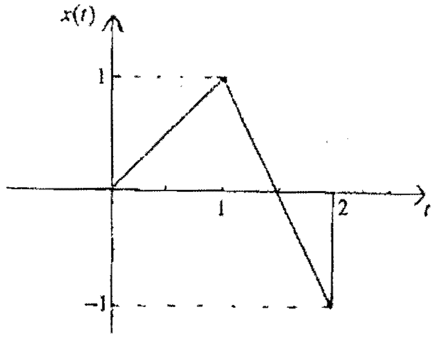

Fig. 1

(ii) (5 pts) Calculate the energy of the signal $x(t)$ given above.

(b) (8pts) Define the following terms.

(i) Linear system

(ii) Causal system

(iii) Dynamic system

(iv) Time-invariant system

(c) (9 pts) Fill in the following table (for each column, give a "yes/no" answer and briefly justify it):

System	Linear	Time-invariant	Causal
$y(n) = 3x(n+1)u(n) - x(n)$			
$y(t) = 2x(t)\cos(t)$			
$y(t) = \int_t^{t+1} x(\tau)d\tau$			

QUESTION TWO *(25 marks)*

(a) (20 pts.) Consider the following circuit (fig. 2):

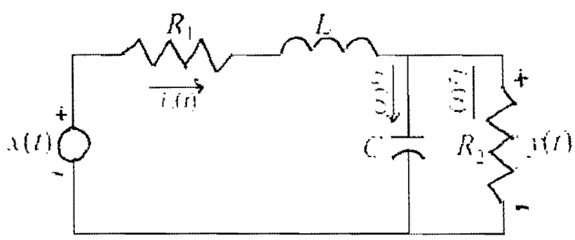

Fig. 2

Write down the input-output differential equation for this circuit in terms of the input voltage $x(t)$ and the output voltage $y(t)$.

(b) (5 pts) Consider a discrete-time system which has input of signal x(n) and output of $y(n) = \cos\left(\frac{\pi}{4}x(n)\right)$. Evaluate and draw the impulse response of the above system.

QUESTION THREE (*25 marks*)

(a) A signal $x(t)$ is passed through an LTI system with impulse response $h(t)$ where

$$x(t) = \begin{cases} 1, & 0 \le t \le 3 \\ 0, & otherwise \end{cases}, \quad \text{and} \quad h(t) = \begin{cases} \beta^n, & 0 \le t \le 5 \\ 0, & otherwise \end{cases}$$

(i) (15 pts) Find expressions for the output signal $y(t)$. The signal $y(t)$ may be divided into clearly defined time intervals.

(ii) (3 pts) Roughly sketch $y(t)$.

(b) (7 pts) The periodic discrete-time signal $x[n]$ has period 4.

$$x(n) = \begin{cases} 1 & \text{for } n = 1 \\ -1 & \text{for } n = 3 \\ 0 & \text{for } n = 0,2 \end{cases}$$

Find the Fourier series coefficients and sketch their magnitudes.

QUESTION FOUR (*25 marks*)

(a) (3+4+5 pts) Find the Laplace transform (LT) of the following continuous-time signals.

(i) t^2

(ii) $\sin(2t)\cos(2t)$

(iii) $te^{-2t}\cos(2t)$

(b) (5+8 pts) Find the inverse Laplace transform of

(i) $F(s) = \frac{3s+5}{s^2+7}$

(ii) $F(s) = \frac{e^{-3s}}{s(s^2+3s+2)}$

QUESTION FIVE *(25 marks)*

(a) (10 pts) Consider a continuous-time LTI system which has impulse response of $h(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & otherwise \end{cases}$. The input of $x(t) = \sum_{k=-1}^{1} \delta(t-4k)$ is applied to this system.

(i) Find the output.

(ii) Draw both input $x(t)$ and output $y(t)$.

(b) (15 pts) Consider the signal $x(t)$ with Fourier transform

$$X(\omega) = \begin{cases} 1, & \text{for } |\omega| \le 20\pi \\ 0, & otherwise \end{cases}.$$

For the following signals, find a mathematical expression for the Fourier transform and plot its magnitude.

(i) $y(t) = 2x(t)cos(15\pi t)$

(ii) $z(t) = x(t-100)$

(iii) $q(t) = x(-\frac{t}{5})$

TABLE 1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\dfrac{\omega\tau}{2}\right)$	
18	$\dfrac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\dfrac{\omega}{2W}\right)$	
19	$\Delta\left(\dfrac{t}{\tau}\right)$	$\dfrac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$	
20	$\dfrac{W}{2\pi}\operatorname{sinc}^2\left(\dfrac{Wt}{2}\right)$	$\Delta\left(\dfrac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\|a\|} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\, du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Table of Laplace Transforms

delta function	$\delta(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	1
shifted delta function	$\delta(t-a)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	e^{-as}
unit step	$u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}$
ramp	$tu(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2}{s^3}$
n-th power	t^n	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\lvert t\rvert}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$-F'(s)$
frequency n-th differentiation	$t^n f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$(-1)^n F^{(n)}(s)$
time differentiation	$f'(t) = \frac{d}{dt}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$sF(s)-f(0)$
time 2nd differentiation	$f''(t) = \frac{d^2}{dt^2}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^2F(s)-sf(0)-f'(0)$
time n-th differentiation	$f^{(n)}(t) = \frac{d^n}{dt^n}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau = (u*f)(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\overset{\mathcal{L}}{\Longleftrightarrow}$	$\frac{F(s)}{s^n}+\frac{f^{-1}(0)}{s^n}+\frac{f^{-2}(0)}{s^{n-1}}+\ldots+\frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

i) Time-shift (delay): $f(t-t_0) \overset{L}{\longleftrightarrow} F(s)e^{-st_0}, \quad t_0 > 0$

ii) Time differentiation: $\dfrac{df(t)}{dt} \overset{L}{\longleftrightarrow} sF(s) - f(0)$

iii) Time integration: $\int_0^t f(t)dt \overset{L}{\longleftrightarrow} \dfrac{F(s)}{s}$

iv) Linearity: $af(t) + bg(t) \overset{L}{\longleftrightarrow} aF(s) + bF(s)$

v) Convolution Integral: $x(t) * h(t) \overset{L}{\longleftrightarrow} X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t} f(t) \overset{L}{\longleftrightarrow} F(s-\alpha)$

vii) Multiplying by t: $tf(t) \overset{L}{\longleftrightarrow} -\dfrac{dF(s)}{ds}$

viii) Scaling: $f(at) \overset{L}{\longleftrightarrow} \dfrac{1}{a} F\left(\dfrac{s}{a}\right), \quad a > 0$

ix) Initial Value Theorem: $\lim_{s\to\infty} \{sF(s)\} = f(0)$

x) Final Value Theorem: $\lim_{s\to 0} \{sF(s)\} = f(\infty)$