

UNIVERSITY OF SWAZILAND
FACULTY OF SCIENCE & ENGINEERING
DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING
SIGNALS AND SYSTEMS I
COURSE CODE – EEE331/EE331
SUPPLEMENTARY EXAMINATION
JULY 2018
DURATION OF THE EXAMINATION - 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. There are **FOUR** questions in this paper. Answer **ALL** the questions.
3. Each question carries 25 marks.
4. Show all your steps clearly in any calculations/work
5. Start each new question on a fresh page.
6. Useful Fourier and Laplace transform properties are **attached**.
7. Make sure that this exam contains 7 pages including this one.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION ONE (25 marks)

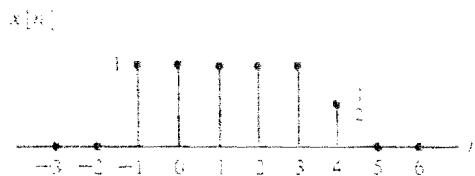
(a) (6 pts.) Define the following terms.

- (i) Linear system
- (ii) Time-invariant system
- (iii) Causal system

(b) (9 pts) Fill in the following table (for each column, give a "yes/no" answer and briefly justify it):

System	Linear	Time-invariant	Causal
$y(t) = x(t+1)u(t) + 1$			
$y(n) = 5x(n)\sin(n)$			
$y(t) = x^2(t-2)$			

(c) (10 pts.) A discrete-time signal $x(n]$ is shown in figure below



- (i) Sketch and carefully label the signal: $x(-n) - x(n-2)$
- (ii) Calculate the energy of the signal $x(n)$ given above.

QUESTION TWO (25 marks)

(a) (10 pts) Given a first-order system

$$y(n) + \frac{1}{2}y(n-1) = 2x(n)$$

Find the system response with initial condition of rest for $x(n) = \left(\frac{-1}{2}\right)^n u(n)$.

Express your result in closed form.

(b) (10 pts) Using Fourier transform, Find $y(t)$, given: $x(t) = e^{\beta t}u(-t)$,

$$h(t) = e^{-\beta t}u(t), \quad \beta > 0.$$

(a) (5 pts) Consider the signal $x(t)$ with Fourier transform

$$X(\omega) = \begin{cases} 1, & \text{for } |\omega| \leq 30\pi \\ 0, & \text{otherwise} \end{cases}$$

Find a mathematical expression for the Fourier transform of $y(t) = 2x(t)\cos(40\pi t)$.

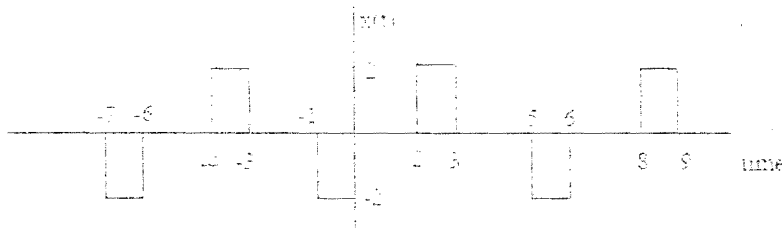
QUESTION THREE (25 marks)

- (a) (15 pts.) A signal $x(t)$ is passed through an LTI system with impulse response $h(t)$ where

$$x(n) = \begin{cases} 1, & 0 \leq n \leq 3 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad h(n) = \begin{cases} \alpha^n, & 0 \leq n \leq 5, \quad 0 < \alpha < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find expressions for the output signal $y(n)$. The signal $y(n)$ may be divided into clearly defined time intervals.

- (b) (10 pts.) Determine the fundamental frequency, period and the coefficients of the Fourier series for the following signal $x(t)$.

**QUESTION FOUR (25 marks)**

- (a) (3+3+3+6 pts) Determine the Laplace transform and associated ROC for each of the following time functions.

(i) $e^{-at}u(t), a < 0$

(ii) t^2

(iii) te^{-3t}

(iv) $te^{-3t}\cos(2t)$

- (b) (5+5 pts) Find the inverse Laplace transform of

(i) $F(s) = \frac{3s+5}{s^2+7}$

(ii) $F(s) = \frac{s}{s(s^2+3s+2)}$

TABLE 4 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\frac{1}{a + j\omega}$	$a > 0$
2	$e^{at}u(-t)$	$\frac{1}{a - j\omega}$	$a > 0$
3	$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	$a > 0$
4	$te^{-at}u(t)$	$\frac{1}{(a + j\omega)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a + j\omega)^{n+1}}$	$a > 0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	
9	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	
10	$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$	
11	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
12	$\text{sgn } t$	$\frac{2}{j\omega}$	
13	$\cos \omega_0 t u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$	
14	$\sin \omega_0 t u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$	
15	$e^{-at} \sin \omega_0 t u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
16	$e^{-at} \cos \omega_0 t u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$	$a > 0$
17	$\text{rect}\left(\frac{t}{\tau}\right)$	$\tau \text{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi} \text{sinc}(Wt)$	$\text{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2} \text{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi} \text{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0)$	$\omega_0 = \frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 2. Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{ a } X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^t x(u) du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Table of Laplace Transforms

delta function	$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1
shifted delta function	$\delta(t-a)$	$\xleftrightarrow{\mathcal{L}}$	e^{-as}
unit step	$u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$
ramp	$tu(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2}{s^3}$
n-th power	t^n	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a t }$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at} \sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at} \cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\xleftrightarrow{\mathcal{L}}$	$-F'(s)$
frequency n-th differentiation	$t^n f(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^n F^{(n)}(s)$
time differentiation	$f'(t) = \frac{d}{dt} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s) - f(0)$
time 2nd differentiation	$f''(t) = \frac{d^2}{dt^2} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2 F(s) - sf(0) - f'(0)$
time n-th differentiation	$f^{(n)}(t) = \frac{d^n}{dt^n} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau) d\tau = (u * f)(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s} F(s)$
frequency integration	$\frac{1}{t} f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u) du$
time inverse	$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)}{s^n} + \frac{f^{-1}(0)}{s^{n-1}} + \frac{f^{-2}(0)}{s^{n-2}} + \dots + \frac{f^{-(n-1)}(0)}{s}$

Properties of Laplace Transforms

- i) Time-shift (delay): $f(t-t_0) \xleftrightarrow{L} F(s)e^{-st_0}$, $t_0 > 0$
- ii) Time differentiation: $\frac{df(t)}{dt} \xleftrightarrow{L} sF(s) - f(0)$
- iii) Time integration: $\int_0^t f(t)dt \xleftrightarrow{L} \frac{F(s)}{s}$
- iv) Linearity: $af(t) + bg(t) \xleftrightarrow{L} aF(s) + bF(s)$
- v) Convolution Integral: $x(t) * h(t) \xleftrightarrow{L} X(s)H(s)$
- vi) Frequency-shift: $e^{at}f(t) \xleftrightarrow{L} F(s-\alpha)$
- vii) Multiplying by t : $tf(t) \xleftrightarrow{L} -\frac{dF(s)}{ds}$
- viii) Scaling: $f(at) \xleftrightarrow{L} \frac{1}{a}F\left(\frac{s}{a}\right)$, $a > 0$
- ix) Initial Value Theorem: $\lim_{s \rightarrow \infty} \{sF(s)\} = f(0)$
- x) Final Value Theorem: $\lim_{s \rightarrow 0} \{sF(s)\} = f(\infty)$