UNIVERSITY OF SWAZILAND

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

SIGNALS AND SYSTEMS II

COURSE CODE – EEE332/EE332

MAIN EXAMINATION

MAY 2018

DURATION OF THE EXAMINATION - 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. There are **FOUR** questions in this paper. Answer **ALL** the questions.

3. Each question carries 25 marks.

4. Show all your steps clearly in any calculations/work

5. Start each new question on a fresh page.

6. Useful Fourier transform and Z-transform properties are **attached**.

7. Make sure that this exam contains 8 pages including this one.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION ONE (*25 marks*)

(a) Using the Fourier transform analysis equation, determine the Fourier transforms of:

(i) $x(n) = \left(\frac{1}{3}\right)^{n-2} u(n-2)$ [pt. 7]

(ii) $x(n) = \left(\frac{1}{3}\right)^{|n-2|}$ [pt. 7]

(b) Consider a causal and stable LTI system S whose input $x(n)$ and output $y(n)$ are related through the second order difference equation

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n).$$

(i) Determine the frequency response $H(e^{j\omega})$ for the system S [pt. 5].

(ii) Determine the impulse response $h(n)$ for the system S [pt. 6].

QUESTION TWO (*25 marks*)

(a) Define the sampling theorem, Nyquist rate and Aliasing. [pt. 4].

(b) Consider the two continuous-time signals

$$x(t) = \sin(\omega_0 t)$$
$$y(t) = x(t) \cdot \cos(\omega_1 t)$$

Where ω_0 and ω_1 are positive finite values. The following signals are sampled using a train of impulses with periodicity T, $\sum_{-\infty}^{+\infty} \delta(t - kT)$: signal $x(t)$ is sampled to obtain $x_c(t)$, signal $y(t)$ is sampled to obtain $y_c(t)$, and signal $x(t) \cdot y(t)$ is sampled to obtain $p_c(t)$.

(i) Determine the range of values for T that allow complete recovery of $x(t)$ from $x_c(t)$ [pt. 5].

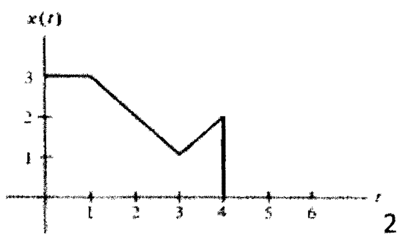
(ii) Determine the range of values for T that allow complete recovery of $y(t)$ from $y_c(t)$ [pt. 8].

(iii) Determine the range of values for T that allow complete recovery of $x(t) \cdot y(t)$ from $p_c(t)$ [pt. 8].

QUESTION THREE (*25 marks*)

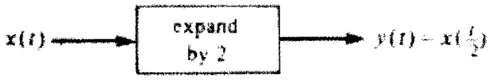
(a) What is amplitude modulation? Consider the signal $x(t)$ shown in figure below.

[pt. 1+6+6]

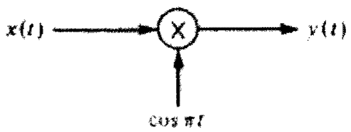


Draw $y(t)$ for each of the following systems.

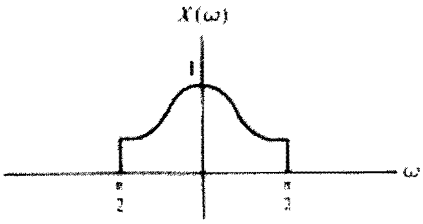
(i)



(ii)



(b) Suppose that $x(t)$ has the Fourier Transform shown in figure below. Find $Y(\omega)$ for each case in part (a). [pt. 6+6]



QUESTION FOUR (*25 marks*)

(a) A digital filter consists of a series of two Linear and Time Invariant (LTI) sub-systems. The z-transform of the unit impulse response of the first sub-system is $H_1(z) = \frac{z}{z-1/2}$ $|z| > 1/2$. The z-transform of the unit impulse response of the second sub-system is $H_2(z) = \frac{z}{z-1}$ $|z| > 1$. Let $h(n)$ be the unit impulse response of the entire digital filter. Derive $H(z)$ and $h(n)$ [pt. 10]

(b) A discrete-time signal x(n) has the z-transform $X(z) = \frac{1}{1+3z^{-1}} \cdot \frac{1}{1+5z^{-1}}$ $|z| > 5$. Determine the z-transform of $y(n) = x(n+2)$ [pt. 8]

(c) Find $x(n)$ from $X(z)$ below using partial fraction expansion, where $x(n)$ is known to be causal, i.e., $x(n) = 0$ for $n < 0$ [pt. 7].

$$X(z) = \frac{3+2z^{-1}}{2+3z^{-1}+z^{-2}}$$

TABLE 1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\lvert t\rvert}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\dfrac{\omega\tau}{2}\right)$	
18	$\dfrac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\dfrac{\omega}{2W}\right)$	
19	$\Delta\left(\dfrac{t}{\tau}\right)$	$\dfrac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$	
20	$\dfrac{W}{2\pi}\operatorname{sinc}^2\left(\dfrac{Wt}{2}\right)$	$\Delta\left(\dfrac{\omega}{2W}\right)$	
21	$\displaystyle\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\displaystyle\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}\,e^{-\sigma^2\omega^2/2}$	

TABLE 2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\, du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

A. Properties of the discrete-time Fourier transform

Non-periodic signal	*Fourier transform*
$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$	$X(e^{j\omega}) \triangleq \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$	$\left.\begin{matrix} X(e^{j\omega}) \\ Y(e^{j\omega}) \end{matrix}\right\}$ Periodic with period 2π
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
$x^*[n]$	$X^*(e^{j(-\omega)})$
$x[-n]$	$X(e^{j(-\omega)})$
$x_{(m)}[n] \doteq \begin{cases} x[n/m], & n \text{ multiple of } m \\ 0, & n \text{ not multiple av } m \end{cases}$	$X(e^{j(m\omega)})$
$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
$x[n] - x[n-1]$	$\left(1 - e^{j\omega}\right) X(e^{j\omega})$
$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-e^{j\omega}}X(e^{j\omega}) + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$
$nx[n]$	$j\frac{d}{d\omega}X(e^{j\omega})$
If $x[n]$ is real valued then	
$x[n]$	$\begin{cases} X(e^{j\omega}) = X^*(e^{j(-\omega)}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{j(-\omega)})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{j(-\omega)})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{j(-\omega)})\rvert \\ \arg\{X(e^{j\omega})\} = -\arg\{X(e^{j(-\omega)})\} \end{cases}$
$x_e[n] = \mathcal{E}\{x[n]\}$	$\Re\{X(e^{j\omega})\}$
$x_o[n] = \mathcal{O}\{x[n]\}$	$j\Im\{X(e^{j\omega})\}$

Parsevals relation for non-periodic signals

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

B. *Discrete-time Fourier transform table*

$x[n]$	$X(e^{j\omega})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$\sum_{k=-\infty}^{\infty} \delta(n-kN)$	$\frac{2\pi}{N}\sum_{k=-\infty}^{\infty} \delta\left(\omega-\frac{2\pi k}{N}\right)$
1	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega-2\pi k)$
$e^{j\omega_o n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\omega-\omega_o-2\pi k)$
$\cos\omega_o n$	$\pi \sum_{k=-\infty}^{\infty} [\delta(\omega-\omega_o-2\pi k)+\delta(\omega+\omega_o-2\pi k)]$
$\sin\omega_o n$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} [\delta(\omega-\omega_o-2\pi k)-\delta(\omega+\omega_o-2\pi k)]$
$u[n]$	$\frac{1}{1-e^{-j\omega}}+\pi \sum_{k=-\infty}^{\infty} \delta(\omega-2\pi k)$
$a^n u(n), \quad \|a\|<1$	$\frac{1}{1-ae^{-j\omega}}$
$(n+1)a^n u[n], \quad \|a\|<1$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{(n+m-1)!}{n!(m-1)!}a^n u[n], \quad \|a\|<1$	$\frac{1}{(1-ae^{-j\omega})^m}$
$\frac{1}{1-a^2}a^{\|n\|}, \quad \|a\|<1$	$\frac{1}{1+a^2-2acos\omega}$
$\begin{cases} 1, & \|n\| \le N_1 \\ 0, & N_1 < \|n\| \le \frac{N}{2} \end{cases}$ period N	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega \frac{2\pi k}{N}\right)$
$\begin{cases} 1, & \|n\| \le N_1 \\ 0, & \|n\| > N_1 \end{cases}$	$\frac{\sin\omega\left(N_1+\frac{1}{2}\right)}{\sin\frac{\omega}{2}}$
$\begin{cases} \frac{\sin Wn}{\pi n} = \frac{W}{\pi}\mathrm{sinc}\frac{Wn}{\pi} \\ 0 < W < \pi \end{cases}$	$\begin{cases} 1, & \|\omega\| \le W \\ 0, & W < \|\omega\| \le \pi \end{cases}$ period 2π

Table of Z-Transforms

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$\lvert z \rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z \rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z \rvert > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z \rvert > 1$
6	$a^nu(n)$	$\frac{z}{z-a}$	$\lvert z \rvert > \lvert a \rvert$
7	$e^{-na}u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z \rvert > e^{-a}$
8	$na^nu(n)$	$\frac{az}{(z-a)^2}$	$\lvert z \rvert > \lvert a \rvert$
9	$\sin(an)u(n)$	$\frac{z\sin(a)}{z^2-2z\cos(a)+1}$	$\lvert z \rvert > 1$
10	$\cos(an)u(n)$	$\frac{z[z-\cos(a)]}{z^2-2z\cos(a)+1}$	$\lvert z \rvert > 1$
11	$a^n\sin(bn)u(n)$	$\frac{[a\sin(b)]z}{z^2-[2a\cos(b)]z+a^2}$	$\lvert z \rvert > \lvert a \rvert$
12	$a^n\cos(bn)u(n)$	$\frac{z[z-a\cos(b)]}{z^2-[2a\cos(b)]z+a^{-2}}$	$\lvert z \rvert > \lvert a \rvert$
13	$e^{-an}\sin(bn)u(n)$	$\frac{[e^{-a}\sin(b)]z}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z \rvert > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\frac{z[z-e^{-a}\cos(b)]}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z \rvert > e^{-a}$

Properties of Z-Transforms

Linearity: $ax_1[k]+bx_2[k] \Leftrightarrow aX_1(z)+bX_2(z)$

Time Reversal: $x[-k] \Leftrightarrow X(1/z)$

Summation: $\sum_{n=-\infty}^{k} x[n] \Leftrightarrow \frac{zX(z)}{z-1}$

Initial Value: $x[0]=\lim_{z\to\infty} X(z)$

Final Value: $x[\infty]=\lim_{z\to 1}(z-1)X(z)$

Convolution: $x[k]*h[k] \Leftrightarrow X(z)H(z)$

Differencing: $x[k]-x[k-1] \Leftrightarrow \left(1-z^{-1}\right)X(z)$

Differentiation: $-kx[k] \Leftrightarrow z\frac{d}{dz}X(z)$

Time Shifting: $x[n-n_o] \Leftrightarrow z^{-n_o}X(z), n_o \geq 0$

$$x[n+n_o] \Leftrightarrow z^{n_o}\left(X(z)-\sum_{m=0}^{n_o-1}x[m]z^{-m}\right), n_o \geq 0$$