UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

SIGNALS AND SYSTEMS II

COURSE CODE – EEE332

MAIN EXAMINATION 2019

DURATION OF THE EXAMINATION - 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Answer all the questions.
2. Each question carries 25 marks.
3. Start each new question on a fresh page.
4. Useful information is attached at the end of the question paper..
5. Make sure that this exam contains 8 pages including this one.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION ONE *(25 marks)*

(a) Determine the Fourier transforms of:

(i) $x(n) = \left(\frac{1}{3}\right)^{n-2} u(n-2)$ [pt. 5]

(ii) $x(n) = 2 + \cos(\frac{\pi}{6}n + \frac{\pi}{8})$ [pt. 5]

(b) A particular LTI system is described by the difference equation

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)$$

Determine the impulse response of the system. [pt. 5]

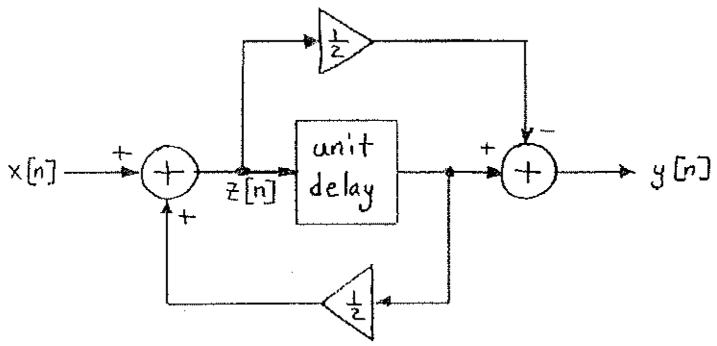
(c) Find the signal corresponding to the following Fourier transforms.

(i) $X(e^{j\omega}) = \frac{3}{1+\frac{1}{4}e^{-j(\omega-\frac{\pi}{4})}}$ [pt. 5]

(ii) $X(e^{j\omega}) = \frac{1-\frac{1}{3}e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}-\frac{1}{8}e^{-j2\omega}}$ [pt. 5]

QUESTION TWO *(25 marks)*

(a) For the LTI discrete-time system shown below:



(i) Find the difference equation from input $x[n]$ to output $z[n]$ (ignore $y[n]$ for this). [pt. 5]

(ii) Find the impulse response $h[n]$ from input $x[n]$ to output $z[n]$. [pt. 10]

(iii) Find the overall impulse response $h_{overall}[n]$ from input $x[n]$ to output $y[n]$. [pt. 5]

(b) Consider the continuous-time signal

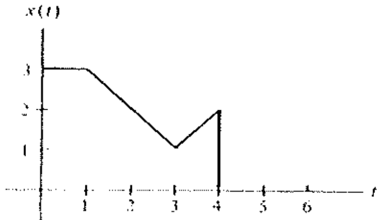
$$x(t) = \frac{\sin(\beta t)}{\pi t}$$

Where β is a positive finite values. Signal $x(t)$ is sampled using a train of impulses with periodicity T, $\sum_{-\infty}^{+\infty} \delta(t - kT)$ to obtain $x_c(t)$.

Determine the range of values for T that allow complete recovery of $x(t)$ from $x_c(t)$. [pt. 5]

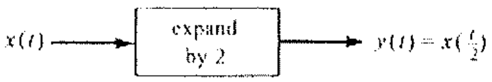
QUESTION THREE *(25 marks)*

(a) What is amplitude modulation? Consider the signal $x(t)$ shown in figure below. [pt. 1+6+6]

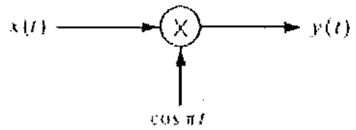


Draw $y(t)$ for each of the following systems.

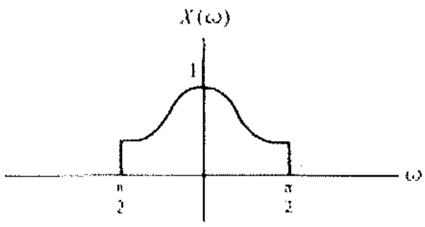
(i)



(ii)



(b) Suppose that $x(t)$ has the Fourier Transform shown in figure below. Find $Y(\omega)$ for each case in part (a). [pt. 6+6]



QUESTION FOUR *(25 marks)*

(a) Determine the z-transform of the following signals: [pt. 3+3]

(i) $x(n) = n(-1)^n u(n)$

(ii) $x(n) = \begin{cases} (\frac{1}{2})^n, & n \geq 5 \\ 0, & n \leq 4 \end{cases}$

(b) Find the inverse z-transform of

$$X(z) = \frac{z}{2z^2 - 3z + 1}, \quad |z| < \frac{1}{2}$$ [pt. 10]

(c) A causal LTI system is described by the difference equation:

$$2y(n) = -3y(n-1) - y(n-2) + x(n-1).$$

Find the system function $H(z)$ and impulse response $h(n)$. [Pt. 9]

Table 1. Continuous Time Fourier transform

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\operatorname{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\dfrac{\omega\tau}{2}\right)$	
18	$\dfrac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\dfrac{\omega}{2W}\right)$	
19	$\Delta\left(\dfrac{t}{\tau}\right)$	$\dfrac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$	
20	$\dfrac{W}{2\pi}\operatorname{sinc}^2\left(\dfrac{Wt}{2}\right)$	$\Delta\left(\dfrac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}\,e^{-\sigma^2\omega^2/2}$	

Table2. CT Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\, du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Table3. Some Trigonometric Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x - \sin^2 x = \cos 2x$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x)$$

Table 4. DTFT properties

DISCRETE-TIME FOURIER TRANSFORM

A. Properties of the discrete-time Fourier transform

Non-periodic signal	*Fourier transform*
$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$	$X(e^{j\omega}) \triangleq \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$
$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$	$\left.\begin{matrix} X(e^{j\omega}) \\ Y(e^{j\omega}) \end{matrix}\right\}$ Periodic with period 2π
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
$x^*[n]$	$X^*(e^{j(-\omega)})$
$x[-n]$	$X(e^{j(-\omega)})$
$x_{(m)}[n] = \begin{cases} x[n/m], & n \text{ multiple of } m \\ 0, & n \text{ not multiple av } m \end{cases}$	$X(e^{j(m\omega)})$
$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
$x[n] - x[n-1]$	$(1 - e^{j\omega})X(e^{j\omega})$
$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-e^{j\omega}}X(e^{j\omega}) + \pi X(0)\sum_{k=-\infty}^{\infty}\delta(\omega - 2\pi k)$
$nx[n]$	$j\frac{d}{d\omega}X(e^{j\omega})$
$x[n]$	*If $x[n]$ is real valued then* $\begin{cases} X(e^{j\omega}) = X^*(e^{j(-\omega)}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{j(-\omega)})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{j(-\omega)})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{j(-\omega)})\rvert \\ \arg\{X(e^{j\omega})\} = -\arg\{X(e^{j(-\omega)})\} \end{cases}$
$x_e[n] = \mathcal{E}\{x[n]\}$	$\Re\{X(e^{j\omega})\}$
$x_o[n] = \mathcal{O}\{x[n]\}$	$j\Im\{X(e^{j\omega})\}$

Parsevals relation for non-periodic signals

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

B. Discrete-time Fourier transform table

$x[n]$	$X(e^{j\omega})$
$\delta[n]$	1
$\delta[n-n_0]$	$e^{-j\omega n_0}$
$\sum_{k=-\infty}^{\infty}\delta(n-kN)$	$\frac{2\pi}{N}\sum_{k=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi k}{N}\right)$
1	$2\pi\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k)$
$e^{j\omega_o n}$	$2\pi\sum_{k=-\infty}^{\infty}\delta(\omega-\omega_o-2\pi k)$
$\cos\omega_o n$	$\pi\sum_{k=-\infty}^{\infty}[\delta(\omega-\omega_o-2\pi k)+\delta(\omega+\omega_o-2\pi k)]$
$\sin\omega_o n$	$\frac{\pi}{j}\sum_{k=-\infty}^{\infty}[\delta(\omega-\omega_o-2\pi k)-\delta(\omega+\omega_o-2\pi k)]$
$u[n]$	$\frac{1}{1-e^{-j\omega}}+\pi\sum_{k=-\infty}^{\infty}\delta(\omega-2\pi k)$
$a^n u(n),\quad \lvert a\rvert<1$	$\frac{1}{1-ae^{-j\omega}}$
$(n+1)a^n u[n],\quad \lvert a\rvert<1$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{(n+m-1)!}{n!(m-1)!}a^n u[n],\quad \lvert a\rvert<1$	$\frac{1}{(1-ae^{-j\omega})^m}$
$\frac{1}{1-a^2}a^{\lvert n\rvert},\quad \lvert a\rvert<1$	$\frac{1}{1+a^2-2acos\omega}$
$\begin{cases}1, & \lvert n\rvert\le N_1\\ 0, & N_1<\lvert n\rvert\le\frac{N}{2}\end{cases}$ period N	$2\pi\sum_{k=-\infty}^{\infty}a_k\delta\left(\omega\frac{2\pi k}{N}\right)$
$\begin{cases}1, & \lvert n\rvert\le N_1\\ 0, & \lvert n\rvert>N_1\end{cases}$	$\frac{\sin\omega\left(N_1+\frac{1}{2}\right)}{\sin\frac{\omega}{2}}$
$\begin{cases}\frac{\sin Wn}{\pi n}=\frac{W}{\pi}\text{sinc}\frac{Wn}{\pi}\\ 0<W<\pi\end{cases}$	$\begin{cases}1, & \lvert\omega\rvert\le W\\ 0, & W<\lvert\omega\rvert\le\pi\end{cases}$ period 2π

Table 5. Z-Transform

Line No.	$x(n), n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$\lvert z\rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z\rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
7	$e^{-an}u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z\rvert > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
9	$\sin(an)u(n)$	$\frac{z\sin(a)}{z^2-2z\cos(a)+1}$	$\lvert z\rvert > 1$
10	$\cos(an)u(n)$	$\frac{z[z-\cos(a)]}{z^2-2z\cos(a)+1}$	$\lvert z\rvert > 1$
11	$a^n\sin(bn)u(n)$	$\frac{[a\sin(b)]z}{z^2-[2a\cos(b)]z+a^2}$	$\lvert z\rvert > \lvert a\rvert$
12	$a^n\cos(bn)u(n)$	$\frac{z[z-a\cos(b)]}{z^2-[2a\cos(b)]z+a^2}$	$\lvert z\rvert > \lvert a\rvert$
13	$e^{-an}\sin(bn)u(n)$	$\frac{[e^{-a}\sin(b)]z}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z\rvert > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\frac{z[z-e^{-a}\cos(b)]}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z\rvert > e^{-a}$
15	$2\lvert A\rvert\lvert P\rvert^n\cos(n\theta+\phi)u(n)$ where P and A are complex constants defined by $P=\lvert P\rvert\angle\theta, A=\lvert A\rvert\angle\phi$	$\frac{Az}{z-P}+\frac{\bar{A}z}{z-\bar{P}}$	

5.4 - Properties of z-Transform

(1) *Linearity :* $ax[n]+by[n] \longleftrightarrow aX(z)+bY(z)$

(2) Time Shifting $x[n-n_0] \longleftrightarrow z^{-n_0}X(z)$,

(3) z-Domain Differentiation $nx[n] \longleftrightarrow -z\frac{dX(z)}{dz}$.

(4) Z-scale Property : $a^n x[n] \longleftrightarrow X\left(\frac{z}{a}\right)$

(5) *Time Reversal :* $x[-n] \longleftrightarrow X\left(\frac{1}{z}\right)$

(6) *Convolution :* $h[n]*x[n] \longleftrightarrow H(z)X(z)$

Transfer Function