University of Eswatini
Faculty of Science and Engineering
Department of Electrical and Electronic Engineering
Main Examination 2019

Title of Paper	:	**Introduction to Digital Signal Processing**
Course Number	:	EEE446/EE443
Time Allowed	:	3 hrs

Instructions :

1. Answer any four (4) questions
2. Each question carries 25 marks
3. Useful information is attached at the end of the question paper
4. Make sure there are 6 pages including the cover page

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QUESTION 1

(a) A digital signal processing (DSP) system is described by the difference equation

$$y(n) - 0.5y(n-1) = 5(0.2)^n u(n)$$

Determine the solution when the initial condition is given by $y(-1) = 1$. (10 marks)

(b) The impulse response $h(n)$, and input $x(n)$ are given as $x(n) = u(n) - u(n-2)$; $h(n) = \left(\frac{1}{3}\right)^n, n \geq 0$. Find the response $y(n)$ in closed form.

(10 marks)

(c) Find the z-transform of $x(n) = n(-1)^n u(n)$ (5 marks)

QUESTION 2

(a) Use the four-point DFT and IDFT to find the sequence

$x_3(n) = x_1(n) \textcircled{N} x_2(n)$, where $x_1(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$ and $x_2(n) = \{\underset{\uparrow}{4}, 3, 2, 2\}$.

(12 marks)

(b) What is the advantage of using fast Fourier transform (FFT) algorithm to determine the DFT of a discrete sequence? Using DIF-FFT algorithm, determine the 8-point DFT of the sequence $x(n) = \{\underset{\uparrow}{1}, 1, 0, 1, 0, 0, 1, 0\}$. (13 marks)

QUESTION 3

(a)

(i) Calculate the filter coefficients for a 5-tap FIR lowpass filter with a cutoff frequency of 800 Hz and a sampling rate of 8,000 Hz using the Hamming window method. (10 marks)

(ii) Determine the transfer function and difference equation of the designed FIR system (5 marks)

(b) Determine the inverse z-transform of

$$X(z)) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}, |z| > 1$$ (5 marks)

(c) Compare the Von Neumann and the Harvard architecture? (5 marks)

QUESTION 4

(a) Given a second-order transfer function

$$H(z) = \frac{0.5(1-0.4z^{-2})}{1+1.4z^{-1}+0.45z^{-2}}$$

Perform the filter realizations and write the difference equations using the following realizations:

(i) Direct form I and direct form II (10 marks)

(ii) Cascade form via the first –order sections (10 marks)

(b) Determine the 4-point DFT of the sequence $x(n) = \{-1, 1, 2\}$ using DIT-FFT.

(5 marks)

QUESTION 5

(a) Design a second-order digital bandpass Butterworth filter with the following specifications: (Use Bilinear transformation method)

- Upper cutoff frequency of 2.6 kHz and
- Lower cutoff frequency of 2.4 kHz,
- Sampling frequency of 8,000 Hz.

State the difference equation and the transfer function of the filter designed

(20 marks)

(b) Use direct Form II to realize the filter. (5 marks)

Table 1: Properties of z-transform

Property	Time Domain	z-Transform
Linearity	$ax_1(n) + bx_2(n)$	$aZ(x_1(n)) + bZ(x_2(n))$
Shift theorem	$x(n-m)$	$z^{-m}X(z)$
Linear convolution	$x_1(n)*x_2(n) = \sum_{k=0}^{\infty} x_1(n-k)x_2(k)$	$X_1(z)X_2(z)$
Differentiation in the z-domain	$nx(n)$	$-z\frac{dX(z)}{dz}$

Table 2: 3 dB Butterworth lowpass prototype transfer functions ($\varepsilon = 1$)

n	$H_P(s)$
1	$\frac{1}{s+1}$
2	$\frac{1}{s^2+1.4142s+1}$
3	$\frac{1}{s^3+2s^2+2s+1}$
4	$\frac{1}{s^4+2.6131s^3+3.4142s^2+2.6131s+1}$
5	$\frac{1}{s^5+3.2361s^4+5.2361s^3+5.2361s^2+3.2361s+1}$
6	$\frac{1}{s^6+3.8637s^5+7.4641s^4+9.1416s^3+7.4641s^2+3.8637s+1}$

Table 3: Summary of ideal impulse responses for standard FIR filters.

Filter Type	Ideal Impulse Response $h(n)$ (noncausal FIR coefficients)
Lowpass:	$h(n) = \begin{cases} \frac{\Omega_c}{\pi} & n = 0 \\ \frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \le n \le M \end{cases}$
Highpass:	$h(n) = \begin{cases} \frac{\pi-\Omega_c}{\pi} & n = 0 \\ -\frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \le n \le M \end{cases}$
Bandpass:	$h(n) = \begin{cases} \frac{\Omega_H-\Omega_L}{\pi} & n = 0 \\ \frac{\sin(\Omega_H n)}{n\pi} - \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \le n \le M \end{cases}$
Bandstop:	$h(n) = \begin{cases} \frac{\pi-\Omega_H+\Omega_L}{\pi} & n = 0 \\ -\frac{\sin(\Omega_H n)}{n\pi} + \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \le n \le M \end{cases}$

Causal FIR filter coefficients: shifting $h(n)$ to the right by M samples.
Transfer function:

$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_{2M} z^{-2M}$
where $b_n = h(n-M)$, $n = 0, 1, \cdots, 2M$

Table 4: FIR filter length estimation using window functions (normalized transition width $\Delta f = |f_{stop} - f_{pass}|/f_s$).

Window Type	Window Function $w(n)$, $-M \leq n \leq M$	Window Length, N	Passband Ripple (dB)	Stopband Attenuation (dB)
Rectangular	1	$N = 0.9/\Delta f$	0.7416	21
Hanning	$0.5 + 0.5\cos\left(\frac{\pi n}{M}\right)$	$N = 3.1/\Delta f$	0.0546	44
Hamming	$0.54 + 0.46\cos\left(\frac{\pi n}{M}\right)$	$N = 3.3/\Delta f$	0.0194	53
Blackman	$0.42 + 0.5\cos\left(\frac{n\pi}{M}\right) + 0.08\cos\left(\frac{2n\pi}{M}\right)$	$N = 5.5/\Delta f$	0.0017	74

Table 5: Analog lowpass prototype transformations

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$

Table 6: Conversion from analog filter specifications to lowpass prototype specifications.

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$v_p = 1$, $v_s = \omega_{as}/\omega_{ap}$
Highpass: ω_{ap}, ω_{as}	$v_p = 1$, $v_s = \omega_{ap}/\omega_{as}$
Bandpass: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$v_p = 1, v_s = \frac{\omega_{ash}-\omega_{asl}}{\omega_{aph}-\omega_{apl}}$
Bandstop: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash} $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$v_p = 1, v_s = \frac{\omega_{aph}-\omega_{apl}}{\omega_{ash}-\omega_{asl}}$

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge; ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband; ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband; ω_o, geometric center frequency.

The Z-transform

Line No.	$x(n),\ n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$\lvert z\rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z\rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert > 1$
6	$a^n u(n)$	$\frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
7	$e^{-an}u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z\rvert > e^{-a}$
8	$na^n u(n)$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
9	$\sin(an)u(n)$	$\frac{z\sin(a)}{z^2-2z\cos(a)+1}$	$\lvert z\rvert > 1$
10	$\cos(an)u(n)$	$\frac{z[z-\cos(a)]}{z^2-2z\cos(a)+1}$	$\lvert z\rvert > 1$
11	$a^n\sin(bn)u(n)$	$\frac{[a\sin(b)]z}{z^2-[2a\cos(b)]z+a^2}$	$\lvert z\rvert > \lvert a\rvert$
12	$a^n\cos(bn)u(n)$	$\frac{z[z-a\cos(b)]}{z^2-[2a\cos(b)]z+a^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
13	$e^{-an}\sin(bn)u(n)$	$\frac{[e^{-a}\sin(b)]z}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z\rvert > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\frac{z[z-e^{-a}\cos(b)]}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z\rvert > e^{-a}$
15	$2\lvert A\rvert\lvert P\rvert^n\cos(n\theta+\phi)u(n)$ where P and A are complex constants defined by $P=\lvert P\rvert\angle\theta,\ A=\lvert A\rvert\angle\phi$	$\frac{Az}{z-P}+\frac{A^*z}{z-P^*}$	