University of Eswatini
Faculty of Science and Engineering
Department of Electrical and Electronic Engineering
Supplementary Exam 2019

Title of Paper : **Introduction to Digital Signal Processing**

Course Number : **EEE446/EE443**

Time Allowed : **3 hrs**

Instructions :

1. Answer all the questions
2. Each question carries 25 marks
3. Useful information is attached at the end of the question paper
4. Make sure there are 6 pages including the cover page

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QUESTION 1

(a) A digital signal processing (DSP) system is described by the difference equation

$$y(n) + 0.6y(n-1) - 0.4y(n-2) = x(n) + x(n-1)$$

Determine the solution when the initial conditions are zero and $x(n) = u(n)$.

(10 marks)

(b) The impulse response $h(n)$, and input $x(n)$ are given as $x(n) = u(n) - u(n-2); h(n) = \left(\frac{1}{3}\right)^n, n \geq 0$. Find the response $y(n)$ in closed form.

(10 marks)

(c) Find the z-transform of $x(n) = (\frac{1}{2})^n[u(n) - u(n-10)]$. (5 marks)

QUESTION 2

(a) Define circular convolution. For the sequences $x_1(n) = \{1, 2, 3, 1\}$ and $x_2(n) = \{4, 3, 2, 2\}$,

Determine the 4-point circular convolution of $x_3(n) = x_1(n) \circledN x_2(n)$.

(10 marks)

(b) What is the advantage of using fast Fourier transform (FFT) algorithm to determine the DFT of a discrete sequence? Using DIT-FFT algorithm, determine the 8-point DFT of the sequence $x(n) = \{1, 1, 0, 1, 1, 0,0,1\}$. What is the speed improvement factor in this case? (15 marks)

QUESTION 3

(a) Design a 5-tap FIR band reject (band-stop) filter with a lower cut-off frequency of 2,000 Hz, an upper cut-off frequency of 2,400 Hz, and a sampling rate of 8,000 Hz using the Hamming window method. Determine the transfer function and difference equation of the desired FIR system.

(20 marks)

(b) Determine the causal signal $x(n)$ having the z-transform

$$X(z)) = \frac{1}{(1+z^{-1})(1-z^{-1})^2}$$ (5 marks)

QUESTION 4

(a) Design a first-order high-pass digital Chebyshev filter with a cut-off frequency of 3 kHz and 1 dB ripple on the pass-band using a sampling frequency of 8,000 Hz. (Use Bilinear Transformation method.)

State the difference equation and the transfer function of the filter designed

(15 marks)

(b) Use direct Form I and II to realize the filter. (10 marks)

Table 1: Properties of z-transform

Property	Time Domain	z-Transform
Linearity	$ax_1(n) + bx_2(n)$	$aZ(x_1(n)) + bZ(x_2(n))$
Shift theorem	$x(n-m)$	$z^{-m}X(z)$
Linear convolution	$x_1(n)*x_2(n) = \sum_{k=0}^{\infty} x_1(n-k)x_2(k)$	$X_1(z)X_2(z)$
Differentiation in the z-domain	$nx(n)$	$-z\dfrac{dX(z)}{dz}$

TABLE 8.5 Chebyshev lowpass prototype transfer functions with 1 dB ripple ($\varepsilon = 0.5088$)

n	$H_P(s)$
1	$\dfrac{1.9652}{s+1.9652}$
2	$\dfrac{0.9826}{s^2+1.0977s+1.1025}$
3	$\dfrac{0.4913}{s^3+0.9883s^2+1.2384s+0.4913}$
4	$\dfrac{0.2456}{s^4+0.9528s^3+1.4539s^2+0.7426s+0.2756}$
5	$\dfrac{0.1228}{s^5+0.9368s^4+1.6888s^3+0.9744s^2+0.5805s+0.1228}$
6	$\dfrac{0.0614}{s^6+0.9283s^5+1.9308s^4+1.2021s^3+0.9393s^2+0.3071s+0.0689}$

Table 3: Summary of ideal impulse responses for standard FIR filters.

Filter Type	Ideal Impulse Response $h(n)$ (noncausal FIR coefficients)
Lowpass:	$h(n) = \begin{cases} \frac{\Omega_c}{\pi} & n = 0 \\ \frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Highpass:	$h(n) = \begin{cases} \frac{\pi-\Omega_c}{\pi} & n = 0 \\ -\frac{\sin(\Omega_c n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Bandpass:	$h(n) = \begin{cases} \frac{\Omega_H-\Omega_L}{\pi} & n = 0 \\ \frac{\sin(\Omega_H n)}{n\pi} - \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$
Bandstop:	$h(n) = \begin{cases} \frac{\pi-\Omega_H+\Omega_L}{\pi} & n = 0 \\ -\frac{\sin(\Omega_H n)}{n\pi} + \frac{\sin(\Omega_L n)}{n\pi} \text{ for } n \neq 0 & -M \leq n \leq M \end{cases}$

Causal FIR filter coefficients: shifting $h(n)$ to the right by M samples.
Transfer function:

$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_{2M} z^{-2M}$
where $b_n = h(n-M)$, $n = 0, 1, \cdots, 2M$

Table 4: FIR filter length estimation using window functions (normalized transition width $\Delta f = |f_{stop} - f_{pass}|/f_s$).

Window Type	Window Function $w(n)$, $-M \leq n \leq M$	Window Length, N	Passband Ripple (dB)	Stopband Attenuation (dB)
Rectangular	1	$N = 0.9/\Delta f$	0.7416	21
Hanning	$0.5 + 0.5\cos\left(\frac{\pi n}{M}\right)$	$N = 3.1/\Delta f$	0.0546	44
Hamming	$0.54 + 0.46\cos\left(\frac{\pi n}{M}\right)$	$N = 3.3/\Delta f$	0.0194	53
Blackman	$0.42 + 0.5\cos\left(\frac{n\pi}{M}\right) + 0.08\cos\left(\frac{2n\pi}{M}\right)$	$N = 5.5/\Delta f$	0.0017	74

Table 5: Analog lowpass prototype transformations

Filter Type	Prototype Transformation
Lowpass	$\frac{s}{\omega_c}$, ω_c is the cutoff frequency
Highpass	$\frac{\omega_c}{s}$, ω_c is the cutoff frequency
Bandpass	$\frac{s^2+\omega_0^2}{sW}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$
Bandstop	$\frac{sW}{s^2+\omega_0^2}$, $\omega_0 = \sqrt{\omega_l \omega_h}$, $W = \omega_h - \omega_l$

Table 6: Conversion from analog filter specifications to lowpass prototype specifications.

Analog Filter Specifications	Lowpass Prototype Specifications
Lowpass: ω_{ap}, ω_{as}	$v_p = 1$, $v_s = \omega_{as}/\omega_{ap}$
Highpass: ω_{ap}, ω_{as}	$v_p = 1$, $v_s = \omega_{ap}/\omega_{as}$
Bandpass: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash}	$v_p = 1, v_s = \frac{\omega_{ash}-\omega_{asl}}{\omega_{aph}-\omega_{apl}}$
$\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	
Bandstop: ω_{apl}, ω_{aph}, ω_{asl}, ω_{ash}	$v_p = 1, v_s = \frac{\omega_{aph}-\omega_{apl}}{\omega_{ash}-\omega_{asl}}$
$\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}$, $\omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge; ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband; ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband; ω_o, geometric center frequency.

The Z-transform

Line No.	$x(n)$, $n \geq 0$	z-Transform $X(z)$	Region of Convergence
1	$x(n)$	$\sum_{n=0}^{\infty} x(n)z^{-n}$	
2	$\delta(n)$	1	$\lvert z\rvert > 0$
3	$au(n)$	$\frac{az}{z-1}$	$\lvert z\rvert > 1$
4	$nu(n)$	$\frac{z}{(z-1)^2}$	$\lvert z\rvert > 1$
5	$n^2u(n)$	$\frac{z(z+1)}{(z-1)^3}$	$\lvert z\rvert > 1$
6	$a^nu(n)$	$\frac{z}{z-a}$	$\lvert z\rvert > \lvert a\rvert$
7	$e^{-na}u(n)$	$\frac{z}{(z-e^{-a})}$	$\lvert z\rvert > e^{-a}$
8	$na^nu(n)$	$\frac{az}{(z-a)^2}$	$\lvert z\rvert > \lvert a\rvert$
9	$\sin(an)u(n)$	$\frac{z\sin(a)}{z^2-2z\cos(a)+1}$	$\lvert z\rvert > 1$
10	$\cos(an)u(n)$	$\frac{z[z-\cos(a)]}{z^2-2z\cos(a)+1}$	$\lvert z\rvert > 1$
11	$a^n\sin(bn)u(n)$	$\frac{[a\sin(b)]z}{z^2-[2a\cos(b)]z+a^2}$	$\lvert z\rvert > \lvert a\rvert$
12	$a^n\cos(bn)u(n)$	$\frac{z[z-a\cos(b)]}{z^2-[2a\cos(b)]z+a^2}$	$\lvert z\rvert > \lvert a\rvert$
13	$e^{-an}\sin(bn)u(n)$	$\frac{[e^{-a}\sin(b)]z}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z\rvert > e^{-a}$
14	$e^{-an}\cos(bn)u(n)$	$\frac{z[z-e^{-a}\cos(b)]}{z^2-[2e^{-a}\cos(b)]z+e^{-2a}}$	$\lvert z\rvert > e^{-a}$
15	$2\lvert A\rvert\lvert P\rvert^n\cos(n\theta+\phi)u(n)$ where P and A are complex constants defined by $P = \lvert P\rvert\angle\theta, A = \lvert A\rvert\angle\phi$	$\frac{Az}{z-P}+\frac{A^*z}{z-P^*}$	