

UNIVERSITY OF ESWATINI



MAIN EXAMINATION PAPER 2018

TITLE OF PAPER : PROBABILITY AND STATISTICS

COURSE CODE : EEE301

TIME ALLOWED : 3 HOURS

INSTRUCTIONS : ANSWER ANY FIVE QUESTIONS.

**REQUIREMENTS : SCIENTIFIC CALCULATOR AND
STATISTICAL TABLES.**

Question 1

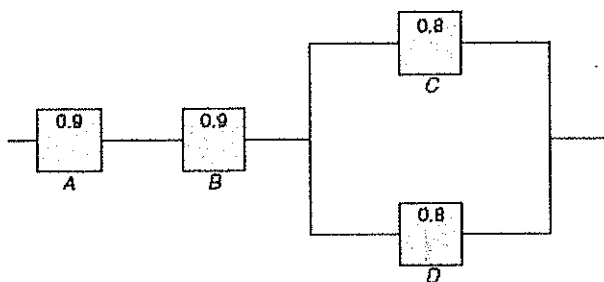
- A cell-phone tower has a circular coverage area of radius 10 Km. If a call is initiated from a random point in the coverage area, find the probability that the call comes from within 2 Km of the tower.
- Consider the duration of a cell-phone call. For the sample space $\Omega = [0, \infty)$, find the probability that the duration is between 5 and 7 minutes.
- Three bits are transmitted across a noisy channel and the number of correct receptions is noted. Find the probability that the number of correctly received bits is two, assuming bit errors are mutually independent and that on each transmission the probability of correct reception is λ for some fixed $0 \leq \lambda \leq 1$.
- The number of hits to a popular website during a 1-minute interval is given by X , random variable. Find the probability that there is at least one hit between 3:00 a.m and 3:01 a.m if rate parameter is 2 hits per minute. Then find the probability that there are at least 2 hits during this time interval.

(5+5+5+5 Marks)

Question 2

- Due to an Internet configuration error, packets sent from Mbabane to University of Eswatini are routed through Ntontozi with probability $3/4$. Given that a packet is routed through Ntontozi, suppose it has probability $1/3$ of being dropped. Given that a packet is not routed through Ntontozi, suppose it has probability $1/4$ of being dropped.
 - Find the probability that a packet is dropped.
 - Find the probability that a packet is routed through Ntontozi when it is not dropped.

- An electrical system consists of four components.



The system works if components A and B work and either of the components C or D works. The reliability (probability of working) of each component is also shown in the graph. Find the probability that

- The entire system works

- ii. component C does not work, given that the entire system works. Assume that the four components work independently.

(5+5 Marks)

Question 3

- a) Let X denote the life time (in hundreds of hours) of a certain type of electronic component. These components frequently fail immediately upon insertion into the system. It has been observed that the probability of immediate failure is 1/4. If a component does not fail immediately, the life-length distribution has the exponential density:

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0 & \text{Otherwise} \end{cases}$$

Find the distribution function for X and evaluate $P(X > 10)$.

(10 Marks)

- b) If X is a random variable characterized by the pdf $f(x) = \lambda e^{-\lambda x}$, $x > 0$ variable with parameter $\lambda = 1$, Show that k^{th} moments of X is $k!$.

(10 Marks)

Question 4

- a) A company prices its hurricane insurance using the following assumptions:
- In any calendar year, there can be at most one hurricane.
 - In any calendar year, the probability of a hurricane is 0.05.
 - The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

(10 Marks)

- b) An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4. What is the expected benefit under this policy?

(10 Marks)

Question 5

- a) The probability function for random variable X is

$$f(x) = \begin{cases} 2^{-x}, & x = 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$$

Find the probability function of a random variable $U = X^4 + 1$.

(10 Marks)

- b) The probability function of a random variable X is given by

$$f(x) = \begin{cases} x^2/18, & -3 < x < 6 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability function for the random variable $U = \frac{1}{3}(12 - X)$.

(10 Marks)

Question 6

A company manufacturing pacemakers is testing a new electrode. The electrodes must adhere to a silicone substrate for at least 20 years. The company is going to test the hypothesis that the mean adherence time is 20 years vs. the alternative that it is less than 20 years at the significance level $\alpha = 0.05$. The experiment will be conducted with a sample of 25 volunteers. Assume that the population distribution for the adherence time is approximately normally distributed.

The average adherence time for the pacemakers in the 25 volunteers is found to be 18.8 years and the standard deviation of the sample is found to be 3 years.

a) Is the null hypothesis rejected?

(10 Marks)

b) If the company wants to decrease the probability of making a type I error without increasing the sample size, should the critical value be increased or decreased? Justify your answer.

(5 Marks)

c) Find the 95% confidence interval for the population variance σ^2 .

(5 Marks)

Question 7

A company manufacturing light bulbs is testing a new model. The company is going to test the hypothesis that the mean life time is 1000 hours vs. the alternative hypothesis that it is less than 1000 hours at the significance level = 0.02. Assume that the population distribution for life time is approximately normal. A sample of 16 light bulbs are found to have sample mean $\bar{x} = 987.5$ hours and sample variance $S^2 = 400$.

a) State the critical region and answer whether the null hypothesis H_0 is rejected.

(12 Marks)

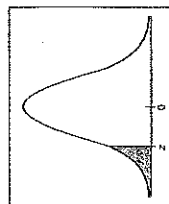
b) Find a 90% confidence interval for the population variance σ^2 .

(8 Marks)

AZZ

Appendix II

Tables



The table entry for z is the area to the left of z.

TABLE 5 Areas of a Standard Normal Distribution.

(a) Table of Areas to the Left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0005	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0008	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0011	.0010	.0010
-2.9	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014	.0014
-2.8	.0025	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0020
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0060	.0058	.0057	.0055	.0054	.0052	.0051	.0049	.0048	.0046
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1786	.1758	.1728	.1701	.1675	.1650	.1625	.1601
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4552	.4502	.4453	.4404	.4354	.4304	.4255	.4206	.4157
-0.0	.5000	.4950	.4900	.4850	.4801	.4751	.4701	.4651	.4601	.4551

For values of z less than -3.48, use 0.000 to approximate the area.

TABLE 5(a) continued

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9991	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

For z value greater than 3.48, use 1.000 to approximate the area.

TABLE 5 continued

(c) Hypothesis Testing, Critical Values z_{α}

Level of Significance	$\alpha = 0.05$	$\alpha = 0.01$
Critical value z_{α} for a left-tailed test	-1.645	-2.33
Critical value z_{α} for a right-tailed test	1.645	2.33
Critical values $\pm z_{\alpha/2}$ for a two-tailed test	± 1.96	± 2.58

TABLE 5 continued

(b) Confidence Interval

Level of Confidence c	Critical Value $z_{\alpha/2}$	Value z_{α}
0.70, or 70%	1.04	1.15
0.75, or 75%	1.15	1.28
0.80, or 80%	1.28	1.44
0.85, or 85%	1.44	1.645
0.90, or 90%	1.645	1.96
0.95, or 95%	1.96	2.33
0.99, or 99%	2.58	2.58

TABLE 8 continued

Degrees of freedom denominator, $d.f.D$	Right tail area	Degrees of freedom numerator, $d.f.N$								
		1	2	3	4	5	6	7	8	9
0.100	3.35	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	
0.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.22	3.18	
9 0.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	
0.010	10.55	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	
0.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11	
0.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	
0.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	
10 0.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	
0.010	10.04	7.56	6.53	5.95	5.64	5.39	5.20	5.06	4.94	
0.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96	
0.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	
0.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	
11 0.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	
0.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	
0.001	19.69	13.81	11.56	10.35	9.58	9.05	8.65	8.35	8.12	
0.100	3.16	2.81	2.61	2.49	2.39	2.33	2.28	2.24	2.21	
0.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	
12 0.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	
0.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	
0.001	18.64	12.97	10.80	9.63	8.86	8.33	8.00	7.71	7.46	
0.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	
0.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	
13 0.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	
0.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	
0.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.96	
0.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	
0.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	
14 0.025	6.30	4.86	4.24	3.88	3.66	3.50	3.38	3.29	3.21	
0.010	8.85	6.51	5.55	5.04	4.69	4.45	4.28	4.14	4.03	
0.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58	
0.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	
0.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	
15 0.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	
0.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	
0.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.25	
0.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	
0.050	4.49	3.63	3.24	3.01	2.85	2.74	2.65	2.59	2.54	
16 0.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	
0.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	
0.001	16.14	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98	