UNIVERSITY OF ESWATINI

MAIN EXAMINATION, FIRST SEMESTER

DECEMBER 2018

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TITLE OF PAPER: CONTROL ENGINEERING I

COURSE CODE : EEE431 / EE431

TIME ALLOWED: THREE HOURS

INSTRUCTIONS:

1. **There are five questions in this paper. Answer any four questions. Each question carries 25 marks.**
2. **Useful information is provided on the last page of this paper.**
3. **If you think not enough data has been given in any question you may assume any reasonable values.**

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THIS PAPER CONTAINS SEVEN (7) PAGES INCLUDING THIS PAGE

Question 1 (25 Marks)

(a) Show that for a system represented in state space the transfer function is
$G(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}$ [10]

(b) For the state space system shown below

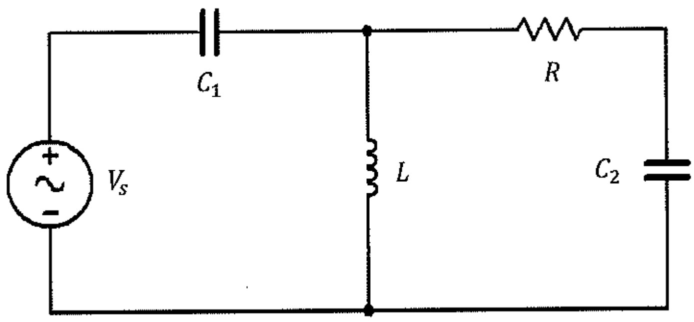
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -361 & -16 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 361 \end{bmatrix} r$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Find the following [15]

(i) Peak time T_p
(ii) Percentage Overshoot (% OS)
(iii) Rising Time (T_r)
(iv) Settling Time (T_s)

Question 2

(a) Find the state space representation of the following electrical network given that the output is the voltage across C_2. [15]



(b) Represent the following transfer function in state space equations and matrix, also show the decomposed transfer function and the equivalent block diagram. [10]

$$T(s) = \frac{s^2+5s+4}{(s+3)(s^2+7s+9)}$$

Question 3 (25 Marks)

(a) Consider a plant with the following transfer function

$$T(s) = \frac{s-2}{(s+3)(S^2+2S+17)}$$

Determine the out time response if the input is a step. [15]

(b) Simplify the following block diagram in Fig. Q.3(b) into a single transfer function using block reduction techniques [10]

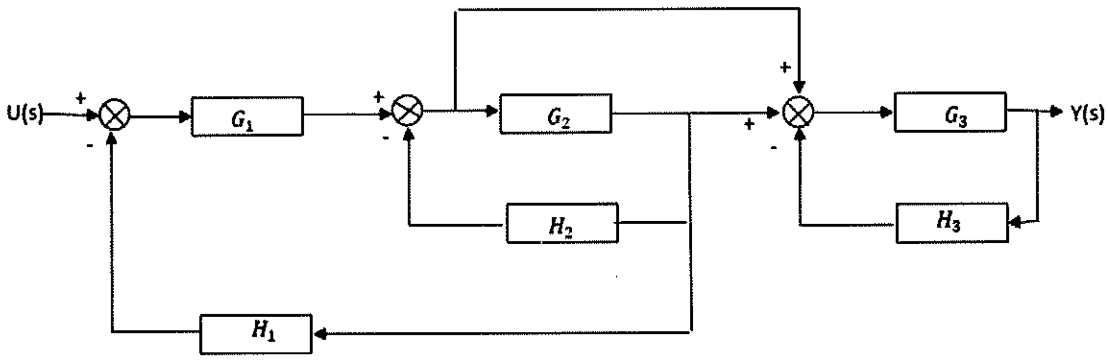


Fig. Q.3 (b)

Question 4 (25 Marks)

(a) Consider the system shown in the Fig.Q.4(a) below, Determine the range of K for stability [10]

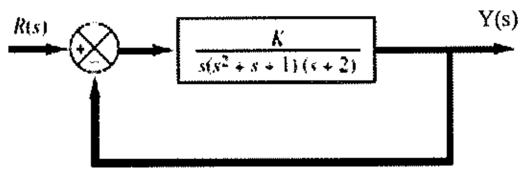


Fig. Q.4 (a)

(b) For the system shown in Fig. 4(b) below show that the proportional control of a system without an integrator will result in a steady-state error with a step input and show that such an error can be eliminated if integral control action is included in the controller. [15]

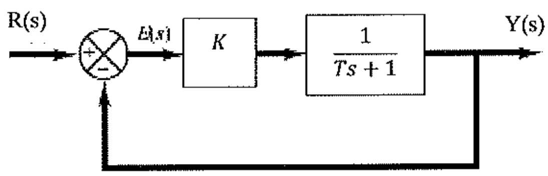


Fig.Q.4 (b)

Question 5 (25 Marks)

(a) Given the system shown in Fig. Q5 (a)

(i) Sketch the root locus, there is no need to annotate break-in/away points and imaginary axis intercepts, if there are any. [10]

(ii) If you had to recommend this system to a customer, what would you advise with respect to increasing the feedback gain K indefinitely? [1]

(iii) Use the angle criteria to determine if the point $s = -5+j3$ is on the root locus of the system described in Fig Q5 (a). [4]

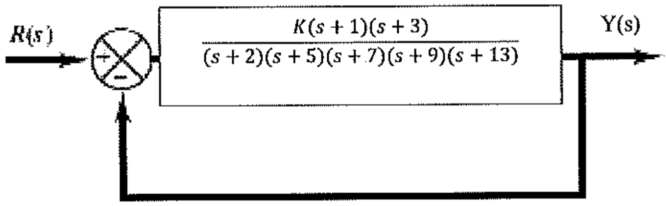


Fig.Q.5 (a)

(b) Discuss the following terms [4]

(i) Gain margin

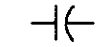
(ii) Phase margin

(c) Calculate the gain margin of system described by the following transfer function. [6]

$$G(s) = \frac{1000}{s(s+5)(s+20)}$$

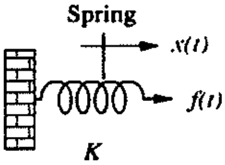
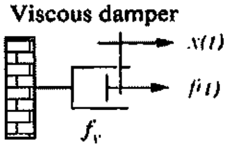
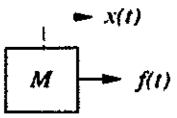
Useful information

Table 1

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ = V (volts), $i(t)$ = A (amps), $q(t)$ = Q (coulombs), C = F (farads), R = Ω (ohms), G = ℧ (mhos), L = H (henries).

Table 2

Component	Force-velocity	Force-displacement	Impedance $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v\frac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

Note: The following set of symbols and units is used throughout this book: $f(t)$ = N (newtons), $x(t)$ = m (meters), $v(t)$ = m/s (meters/second), K = N/m (newtons/meter), f_v = N-s/m (newton-seconds/meter), M = kg (kilograms = newton-seconds^2/meter).

Table 3

Input	Steady-state error formula	Type 0		Type 1		Type 2	
		Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

Static Error Constants

For a step input, $u(t)$,

$$e(\infty) = e_{\text{step}}(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

For a ramp input, $tu(t)$,

$$e(\infty) = e_{\text{ramp}}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)}$$

For a parabolic input, $\frac{1}{2}t^2u(t)$,

$$e(\infty) = e_{\text{parabola}}(\infty) = \frac{1}{\lim_{s \to 0} s^2G(s)}$$

Position constant, K_p, where

$$K_p = \lim_{s \to 0} G(s)$$

Velocity constant, K_v, where

$$K_v = \lim_{s \to 0} sG(s)$$

Acceleration constant, K_a, where

$$K_a = \lim_{s \to 0} s^2G(s)$$

$$f^*(t) = \sum_{k=0}^{\infty} kT\delta(t - kT)$$

$$F^*(s) = \sum_{k=0}^{\infty} kTe^{-kTs}$$

$$e^{-kTs} = Z^{-k}$$

Table 4

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	e^{at}	$\frac{1}{s-a}$
3.	$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\ n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$e^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$e^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n e^{at},\ n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ <u>**Heaviside Function**</u>	$\frac{e^{-cs}}{s}$	26.	$\delta(t-c)$ <u>**Dirac Delta Function**</u>	e^{-cs}
27.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$	28.	$u_c(t)g(t)$	$e^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$e^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T e^{-st}f(t)\,dt}{1-e^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			