



UNIVERSITY OF ESWATINI

FIRST SEMESTER RESIT EXAMINATION PAPER, JANUARY 2020

FACULTY OF SCIENCE AND ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

COURSE CODE: EEE301

TITLE OF PAPER: PROBABILITY AND STATISTICS

TIME ALLOWED: 3 HOURS

Instructions

1. This paper consists of five (5) questions
2. Answer any three questions.

Special Requirements

Scientific calculator

Additional Material (s)

1. Statistical Tables

*Candidates may complete the front cover of their answer book when instructed by the Chief Invigilator and sign their examination attendance planed but must **NOT** write anything else until the start of the examination period is announced.*

No electronic devices capable of storing and retrieving text, including electronic dictionaries and any form of foreign material may be used while in the examination room.

DO NOT turn examination paper over until instructed to do so.

Question 1

In a study conducted by the Department of Mechanical Engineering at a university, the steel rods supplied by two different companies were compared. Ten sample springs were made out of the steel rods supplied by each company and a measure of flexibility was recorded for each. The data are as follows:

Company A: 9.3 8.8 6.8 8.7 8.5 6.7 8.0 6.5 9.2 7.0

Company B: 11.0 9.8 9.9 10.2 10.1 9.7 11.0 11.1 10.2 9.6

- a) Calculate the sample mean, median, and variance for the data for the two companies.
(4+4+4 Marks)
- b) Calculate the coefficient of variation for the two companies and comment.
(8 Marks)

Question 2

- a) Interest centres on the life of an electronic component. Suppose it is known that the probability that the component survives for more than 6000 hours is 0.42. Suppose also that the probability that the component survives no longer than 4000 hours is 0.04.
- (i) What is the probability that the life of the component is less than or equal to 6000 hours?
- (ii) What is the probability that the life is greater than 4000 hours?
(3+3 Marks)
- b) A regional telephone company operates three identical relay stations at different locations. During a one year period, the number of malfunctions reported by each station and the causes are shown below.

<u>Station</u>	<u>A</u>	<u>B</u>	<u>C</u>	
Problems with electricity supplied		2	1	1
Computer malfunction	4	3	2	
Malfunctioning electrical equipment	5	4	2	
Caused by other human errors	7	7	5	

Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from station C?

(14Marks)

Question 3

Magnetron tubes are produced from an automated assembly line. A sampling plan is used periodically to assess quality on the lengths of the tubes. This measurement is subject to

uncertainty. It is thought that the probability that a random tube meets length specification is 0.99. A sampling plan is used in which the lengths of 5 random tubes are measured.

- a) Show that the probability function of Y , the number out of 5 that meet length specification, is given by the binomial discrete probability function. (10 Marks)
- b) Suppose random selections are made off the line and 3 are outside specifications. Use probability function above either to support or refute the conjecture that the probability is 0.99 that a single tube meets specifications. (10 Marks)

Question 4

If a dealer's profit, in units of \$5000, on a new automobile can be looked upon as a random variable X having the density function

$$f(x) = 2(1 - x), \quad 0 < x < 1$$

- a) Find the average profit per automobile. (4 Marks)
- b) What is the dealer's average profit and standard deviation per automobile if the profit on each automobile is given by $g(X) = X^2$. (6+5+5 Marks)

Question 5

- a) Derive the mean and variance of a probability density function of this form;

$$f(x; \alpha, \beta) = \beta x^{\beta-1} e^{-\alpha x^\beta}, \quad x > 0$$

(10 Marks)

- b) Suppose that the service life, in years, of a hearing aid battery is a random variable having a Weibull distribution with $\alpha = 1/2$ and $\beta = 2$.

- (i) How long can such a battery be expected to last?
- (ii) What is the probability that such a battery will be operating after 2 years? (5+5 Marks)

Question 6

- a) A machine is producing metal pieces that are cylindrical in shape. A sample of pieces is taken and the diameters are 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, and 1.03 centimetres. Find a 99% confidence interval for the mean diameter and variance of pieces from this machine, assuming an approximate normal distribution. (10 Marks)

- b) The following measurements were recorded for the drying time, in hours, of a certain brand of latex paint:

3.4 2.5 4.8 2.9 3.6 2.8 3.3 5.6 3.7 2.8 4.4 4.0 5.2
3.0 4.8

Assuming that the measurements represent a random sample from a normal population, find the 99% tolerance limits that will contain 95% of the drying times.
(10 Marks)

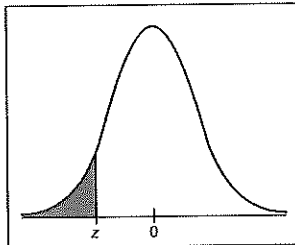
Question 7

A random sample of $n_1 = 32$ specimens of cold-rolled steel give average strength $\bar{X}_1 = 29.8 \text{ ksi}$, and sample standard deviation of $S_1 = 4$. A random sample of $n_2 = 35$ specimens of two-sided galvanized steel give average $\bar{X}_2 = 34.7 \text{ ksi}$, and $S_2 = 6.74$.

- a) Find the 99% confidence interval for $\mu_1 - \mu_2$ (8 Marks)
- b) Are the strengths of the two types of steel different at $\alpha = 0.01$. (12 Marks)

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Appendix II Tables



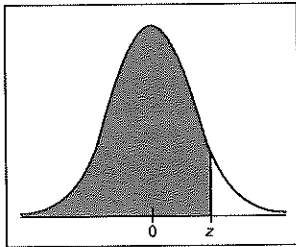
The table entry for z is the area to the left of z .

TABLE 5 Areas of a Standard Normal Distribution

(a) Table of Areas to the Left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

For values of z less than -3.49 , use 0.000 to approximate the area.



The table entry for z is the area to the left of z .

TABLE 5(a) *continued*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

For z values greater than 3.49, use 1.000 to approximate the area.

TABLE 5 *continued*

(b) Confidence Interval Critical Values z_c	
Level of Confidence c	Critical Value z_c
0.70, or 70%	1.04
0.75, or 75%	1.15
0.80, or 80%	1.28
0.85, or 85%	1.44
0.90, or 90%	1.645
0.95, or 95%	1.96
0.98, or 98%	2.33
0.99, or 99%	2.58

TABLE 5 *continued*

(c) Hypothesis Testing, Critical Values z_0		
Level of Significance	$\alpha = 0.05$	$\alpha = 0.01$
Critical value z_0 for a left-tailed test	-1.645	-2.33
Critical value z_0 for a right-tailed test	1.645	2.33
Critical values $\pm z_0$ for a two-tailed test	± 1.96	± 2.58

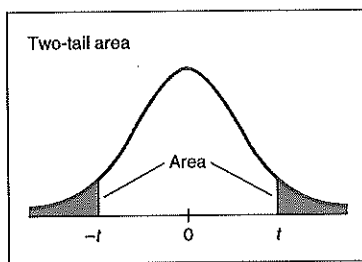
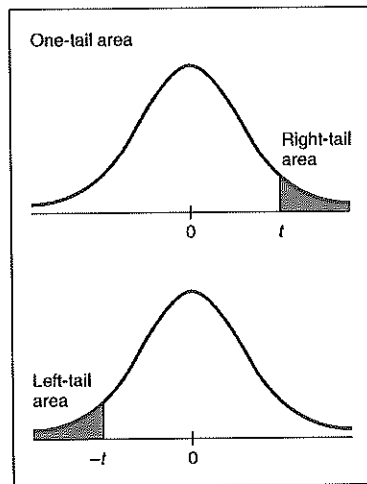
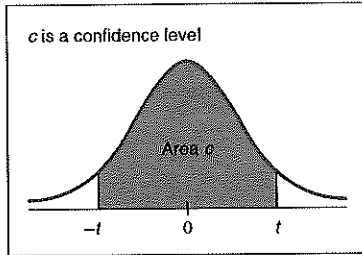


TABLE 6 Critical Values for Student's t Distribution

one-tail area	0.250	0.125	0.100	0.075	0.050	0.025	0.010	0.005	0.0005
two-tail area	0.500	0.250	0.200	0.150	0.100	0.050	0.020	0.010	0.0010
d.f. \ c	0.500	0.750	0.800	0.850	0.900	0.950	0.980	0.990	0.999
1	1.000	2.414	3.078	4.165	6.314	12.706	31.821	63.657	636.619
2	0.816	1.604	1.886	2.282	2.920	4.303	6.965	9.925	31.599
3	0.765	1.423	1.638	1.924	2.353	3.182	4.541	5.841	12.924
4	0.741	1.344	1.533	1.778	2.132	2.776	3.747	4.604	8.610
5	0.727	1.301	1.476	1.699	2.015	2.571	3.365	4.032	6.869
6	0.718	1.273	1.440	1.650	1.943	2.447	3.143	3.707	5.959
7	0.711	1.254	1.415	1.617	1.895	2.365	2.998	3.499	5.408
8	0.706	1.240	1.397	1.592	1.860	2.306	2.896	3.355	5.041
9	0.703	1.230	1.383	1.574	1.833	2.262	2.821	3.250	4.781
10	0.700	1.221	1.372	1.559	1.812	2.228	2.764	3.169	4.587
11	0.697	1.214	1.363	1.548	1.796	2.201	2.718	3.106	4.437
12	0.695	1.209	1.356	1.538	1.782	2.179	2.681	3.055	4.318
13	0.694	1.204	1.350	1.530	1.771	2.160	2.650	3.012	4.221
14	0.692	1.200	1.345	1.523	1.761	2.145	2.624	2.977	4.140
15	0.691	1.197	1.341	1.517	1.753	2.131	2.602	2.947	4.073
16	0.690	1.194	1.337	1.512	1.746	2.120	2.583	2.921	4.015
17	0.689	1.191	1.333	1.508	1.740	2.110	2.567	2.898	3.965
18	0.688	1.189	1.330	1.504	1.734	2.101	2.552	2.878	3.922
19	0.688	1.187	1.328	1.500	1.729	2.093	2.539	2.861	3.883
20	0.687	1.185	1.325	1.497	1.725	2.086	2.528	2.845	3.850
21	0.686	1.183	1.323	1.494	1.721	2.080	2.518	2.831	3.819
22	0.686	1.182	1.321	1.492	1.717	2.074	2.508	2.819	3.792
23	0.685	1.180	1.319	1.489	1.714	2.069	2.500	2.807	3.768
24	0.685	1.179	1.318	1.487	1.711	2.064	2.492	2.797	3.745
25	0.684	1.178	1.316	1.485	1.708	2.060	2.485	2.787	3.725
26	0.684	1.177	1.315	1.483	1.706	2.056	2.479	2.779	3.707
27	0.684	1.176	1.314	1.482	1.703	2.052	2.473	2.771	3.690
28	0.683	1.175	1.313	1.480	1.701	2.048	2.467	2.763	3.674
29	0.683	1.174	1.311	1.479	1.699	2.045	2.462	2.756	3.659
30	0.683	1.173	1.310	1.477	1.697	2.042	2.457	2.750	3.646
35	0.682	1.170	1.306	1.472	1.690	2.030	2.438	2.724	3.591
40	0.681	1.167	1.303	1.468	1.684	2.021	2.423	2.704	3.551
45	0.680	1.165	1.301	1.465	1.679	2.014	2.412	2.690	3.520
50	0.679	1.164	1.299	1.462	1.676	2.009	2.403	2.678	3.496
60	0.679	1.162	1.296	1.458	1.671	2.000	2.390	2.660	3.460
70	0.678	1.160	1.294	1.456	1.667	1.994	2.381	2.648	3.435
80	0.678	1.159	1.292	1.453	1.664	1.990	2.374	2.639	3.416
100	0.677	1.157	1.290	1.451	1.660	1.984	2.364	2.626	3.390
500	0.675	1.152	1.283	1.442	1.648	1.965	2.334	2.586	3.310
1000	0.675	1.151	1.282	1.441	1.646	1.962	2.330	2.581	3.300
∞	0.674	1.150	1.282	1.440	1.645	1.960	2.326	2.576	3.291

For degrees of freedom d.f. not in the table, use the closest d.f. that is smaller.

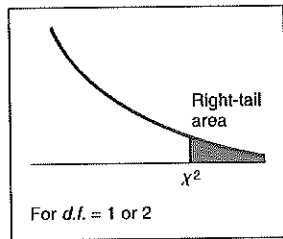
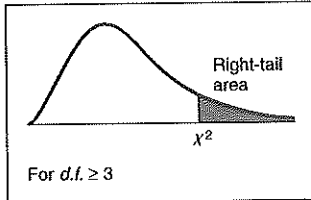


TABLE 7 The χ^2 Distribution

d.f.	Right-tail Area									
	.995	.990	.975	.950	.900	.100	.050	.025	.010	.005
1	0.0 ⁴ 393	0.0 ³ 157	0.0 ³ 982	0.0 ² 393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	8.99	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.21	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: From H. L. Herter, *Biometrika*, June 1964. Printed by permission of the Biometrika Trustees.