UNIVERSITY OF ESWATINI

FACULTY OF SCIENCE & ENGINEERING

DEPARTMENT OF ELECTRICAL & ELECTRONIC ENGINEERING

SIGNALS AND SYSTEMS I

COURSE CODE – EEE331

RESIT EXAMINATION

JANUARY 2020

DURATION OF THE EXAMINATION - 3 HOURS

INSTRUCTIONS TO CANDIDATES

1. Answer all the questions.
2. Each question carries 25 marks.
3. Start each new question on a fresh page.
4. Useful information is attached at the end of the question paper.
5. Make sure that this exam contains 7 pages including this one.

DO NOT OPEN THIS PAPER UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION ONE (*25 marks*)

(a) (6 pts.) Determine whether or not each of the following signals is periodic. If the signal is periodic, determine its fundamental period.

(i) $x(t) = \varepsilon v\{\sin(4\pi t)\, u(t)\}$

(ii) $x(n) = \cos\left(\frac{\pi}{2}n\right)\cos(\frac{\pi}{4}n)$

(b) (10 pts) Determine and sketch the even and odd parts of the signal given in Fig. 1.

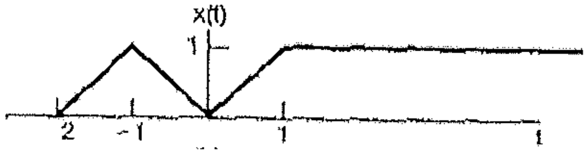


Fig. 1

(c) (9 pts) Determine if each of the following systems is Linear, Time-invariant and/or causal.

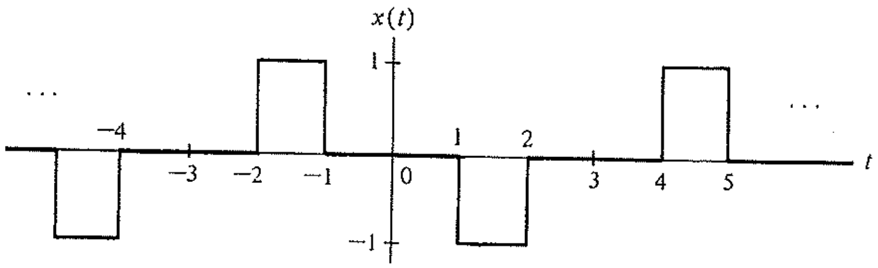
(i) $y(t) = x(t-1)+1$

(ii) $y(n) = x(4n+1)$

(iii) $y(t) = tx(2t)$

QUESTION TWO (*25 marks*)

(a) (10 pts) By evaluating the Fourier series analysis equation, determine the Fourier series for the following signal. Also determine the value of a_0.



(b) (15 pts) Consider the following circuit (fig. 2):

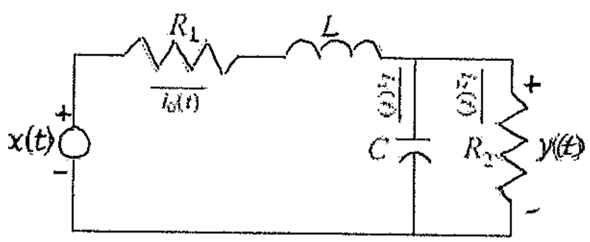


Fig. 2

Write down the input-output differential equation for this circuit in terms of the input voltage $x(t)$ and the output voltage $y(t)$. (Note that $i_c(t) = C\frac{dV_c(t)}{dt}$ and $v_L(t) = L\frac{di_L(t)}{dt}$.)

QUESTION THREE (*25 marks*)

(a) (10 pts.) Find $y(t)$, given:

$$x(t) = e^{\beta t}u(-t),\ \beta > 0, h(t) = e^{-\beta t}u(t).$$

(b) (5 pts) The output of a causal LTI system is related to the input x(t) by the differential equation

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

(i) Determine the frequency response $H(w)$.

(ii) If $x(t) = e^{-t}\,u(t)$, determine $Y(w)$, the Fourier transform of the output and $y(t)$.

(c) (10 pts) Using the definition of the Fourier transform, compute the Fourier transform of: $x(t) = e^{3t}\sin(2t)\,u(-t)$

QUESTION FOUR (*25 marks*)

(a) (12 pts) Determine the Laplace transform and the ROC for each of the following signals:

(i) $x(t) = e^{at}u(t-3)$

Use the definition of Laplace transform. Sketch the pole – zero plot.

(ii) $x(t) = te^{at}u(t) + e^{at}u(t-4) + u(t-5) + 2\delta(t)$

(iii) $x(t) = \sin(3t)\cos(3t)$

(b) (5+8 pts) Find the inverse Laplace transform of

(i) $F(s) = \frac{3s+5}{s^2+7}$

(ii) $F(s) = \frac{s^2+6s+7}{s^2+3s+2}$ Re{s} > -1

Table of Laplace Transforms

delta function	$\delta(t)$	$\xleftrightarrow{\mathcal{L}}$	1
shifted delta function	$\delta(t-a)$	$\xleftrightarrow{\mathcal{L}}$	e^{-as}
unit step	$u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}$
ramp	$tu(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s^2}$
parabola	$t^2u(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2}{s^3}$
n-th power	t^n	$\xleftrightarrow{\mathcal{L}}$	$\frac{n!}{s^{n+1}}$
exponential decay	e^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s+a}$
two-sided exponential decay	$e^{-a\|t\|}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{2a}{a^2-s^2}$
	te^{-at}	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{(s+a)^2}$
	$(1-at)e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{(s+a)^2}$
exponential approach	$1-e^{-at}$	$\xleftrightarrow{\mathcal{L}}$	$\frac{a}{s(s+a)}$
sine	$\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{s^2-\omega^2}$
hyperbolic cosine	$\cosh(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s}{s^2-\omega^2}$
exponentially decaying sine	$e^{-at}\sin(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{\omega}{(s+a)^2+\omega^2}$
exponentially decaying cosine	$e^{-at}\cos(\omega t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{s+a}{(s+a)^2+\omega^2}$
frequency differentiation	$tf(t)$	$\xleftrightarrow{\mathcal{L}}$	$-F'(s)$
frequency n-th differentiation	$t^n f(t)$	$\xleftrightarrow{\mathcal{L}}$	$(-1)^n F^{(n)}(s)$
time differentiation	$f'(t) = \frac{d}{dt}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$sF(s) - f(0)$
time 2nd differentiation	$f''(t) = \frac{d^2}{dt^2}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^2F(s) - sf(0) - f'(0)$
time n-th differentiation	$f^{(n)}(t) = \frac{d^n}{dt^n}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
time integration	$\int_0^t f(\tau)d\tau = (u * f)(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{1}{s}F(s)$
frequency integration	$\frac{1}{t}f(t)$	$\xleftrightarrow{\mathcal{L}}$	$\int_s^\infty F(u)du$
time inverse	$f^{-1}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)-f^{-1}}{s}$
time differentiation	$f^{-n}(t)$	$\xleftrightarrow{\mathcal{L}}$	$\frac{F(s)}{s^n} + \frac{f^{-1}(0)}{s^n} + \frac{f^{-2}(0)}{s^{n-1}} + \ldots + \frac{f^{-n}(0)}{s}$

Properties of Laplace Transforms

i) Time-shift (delay): $f(t-t_0)\overset{L}{\longleftrightarrow}F(s)e^{-st_0}, \quad t_0>0$

ii) Time differentiation: $\dfrac{df(t)}{dt}\overset{L}{\longleftrightarrow}sF(s)-f(0)$

iii) Time integration: $\int_0^t f(t)dt\overset{L}{\longleftrightarrow}\dfrac{F(s)}{s}$

iv) Linearity: $af(t)+bg(t)\overset{L}{\longleftrightarrow}aF(s)+bF(s)$

v) Convolution Integral: $x(t)*h(t)\overset{L}{\longleftrightarrow}X(s)H(s)$

vi) Frequency-shift: $e^{\alpha t}f(t)\overset{L}{\longleftrightarrow}F(s-\alpha)$

vii) Multiplying by t: $tf(t)\overset{L}{\longleftrightarrow}-\dfrac{dF(s)}{ds}$

viii) Scaling: $f(at)\overset{L}{\longleftrightarrow}\dfrac{1}{a}F\left(\dfrac{s}{a}\right), \quad a>0$

ix) Initial Value Theorem: $\lim_{s\to\infty}\{sF(s)\}=f(0)$

x) Final Value Theorem: $\lim_{s\to 0}\{sF(s)\}=f(\infty)$

TABLE 1 Fourier Transforms

No.	$x(t)$	$X(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a > 0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a > 0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a > 0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a > 0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$	
12	$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\dfrac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\dfrac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\dfrac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\dfrac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\dfrac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
17	$\operatorname{rect}\left(\dfrac{t}{\tau}\right)$	$\tau\operatorname{sinc}\left(\dfrac{\omega\tau}{2}\right)$	
18	$\dfrac{W}{\pi}\operatorname{sinc}(Wt)$	$\operatorname{rect}\left(\dfrac{\omega}{2W}\right)$	
19	$\Delta\left(\dfrac{t}{\tau}\right)$	$\dfrac{\tau}{2}\operatorname{sinc}^2\left(\dfrac{\omega\tau}{4}\right)$	
20	$\dfrac{W}{2\pi}\operatorname{sinc}^2\left(\dfrac{Wt}{2}\right)$	$\Delta\left(\dfrac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\dfrac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table 2 Fourier Transform Operations

Operation	$x(t)$	$X(\omega)$
Scalar multiplication	$kx(t)$	$kX(\omega)$
Addition	$x_1(t) + x_2(t)$	$X_1(\omega) + X_2(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Scaling (a real)	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Frequency shifting (ω_0 real)	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi} X_1(\omega) * X_2(\omega)$
Time differentiation	$\frac{d^n x}{dt^n}$	$(j\omega)^n X(\omega)$
Time integration	$\int_{-\infty}^{t} x(u)\, du$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

Useful Formulae:-

Trigonometric Identity:- $sin^2(\theta) = \frac{1}{2}[1 - \cos(2\theta)]$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

Euler's relation:- $e^{j\theta} = \cos(\theta) + jsin(\theta)$

$$Cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right), \quad sin(\theta) = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$