University of Eswatini
Faculty of Science and Engineering
Department of Electrical and Electronic Engineering

Main Examination 2020

Title of Paper:	Modern Control Engineering
Course Number:	EEE533 /EE 532

Time Allowed: 3 hrs

Instructions:

1. Answer all four (4) questions.
2. Each question carries 25 marks.

This paper should not be opened until permission has been given by the invigilator.

This paper contains four (4) pages including this page.

Question 1

a) Derive the modal canonical form of the following system and show the block diagram (10)

$$\frac{Y(s)}{U(s)} = \frac{100(s+10)}{s(s+3)(s+12)}$$

b) From observation of the modal form representation, is the system controllable? (3)
c) Use the controllability matrix approach to confirm your observation (6)
d) Explain what is meant by the following terms for LTI systems.
   i) Controllability (3)
   ii) Observability (3)

Question 2

a) Determine if the following system is stable. (5)

$$\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

b) Use the pole-placement method to design a controller that will place the closed-loop poles of the systems on (a) at $s = -5, -2\ldots.$ (8)

c) For the same system (a), design the controller $k = R^{-1}B^TP$ for which the cost function $J = \int_0^\infty (x^TQx + u^TRu)\ dt$ is minimized, subject to $\dot{x} = Ax + Bu$ (8)

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad R = 1$$

d) List one advantage and one disadvantage of using an optimization method instead of the pole placement approach to design the full state feedback controller $u = -Kx$ (4)

Question 3

a) Define the following terms:
   i) Lyapunov stability (2)
   ii) Local stability (2)
   iii) Global stability (2)

b) Use Lyapunov's indirect method to determine the stability of the following system, for the equilibrium point $x = 0$. (8)

$$\dot{x_1} = -x_2 - \mu x_1(x_1^2 + x_2^2)$$

$$\dot{x_2} = -x_1 - \mu x_2(x_1^2 + x_2^2)$$

c)

i) Determine the equilibrium point for the following system (2)

$$\dot{x_1} = -x_1 + 4x_2$$
$$\dot{x_2} = -x_1 - x_2^3$$

ii) If the system's Lyapunov function is $V = x_1^2 + 4x_2^2$, state the condition for global asymptotic stability (3)

iii) Verify if the system meets this condition (6)

Question 4

a)

i) For the system of figure 1, find the sampled-data transfer function $G(z) = \frac{Y(Z)}{R(Z)}$ if the sampling time, T = 1 second. (12)

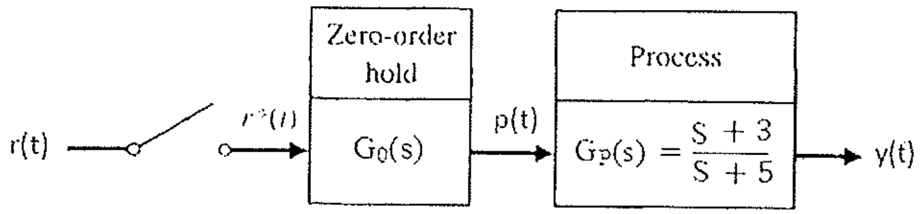

Figure 1

ii) Determine if the closed-loop system is stable for K = 1. (6)

b) Determine the range of sampling time for which the closed-loop system below is stable. (7)

$$G_{CL} = \frac{1-e^{-T}}{z - 7e^{-T} + 5}$$

APPENDIX

z-Transforms

$x(t)$	$X(s)$	$X(z)$
$u(t)$, unit step	$1/s$	$\frac{z}{z-1}$
t	$1/s^2$	$\frac{Tz}{(z-1)^2}$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$1-e^{-at}$	$\frac{1}{s(s+a)}$	$\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$\frac{z(z-\cos(\omega T))}{z^2-2z\cos(\omega T)+1}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{(ze^{-aT}\sin(\omega T))}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{z^2-ze^{-aT}\cos(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$