

**University of Eswatini**  
**Faculty of Science and Engineering**  
**Department of Electrical and Electronic Engineering**

**Main Examination 2021**

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**Title of Paper:** Control Engineering II

**Course Number:** EEE533

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**Time Allowed:** 3 hrs

**Instructions:**

1. Answer all four (4) questions.
2. Each question carries 25 marks.

**This paper should not be opened until permission has been given by the invigilator.**

**This paper contains four (4) pages including this page.**

### Question 1

- a) Show the block diagram of a state-space system with full state-feedback, a state observer and reference input. And briefly describe the design procedure for a controller/observer in state-space systems.

[10]

b)

- i) Convert the following transfer function into phase-variable form

[8]

$$G(s) = \frac{10}{(s^2 + 2)(s + 100)}$$

- ii) Determine if it is observable.

[7]

### Question 2

- a) Discuss 4 common behaviours of non-linear systems.

[8]

- b) For the system  $\dot{x} = Ax$ , using the Lyapunov equation  $AP + A^T P = -Q$ , determine the stability of the system. Use the identity matrix as Q.

[7]

$$A = \begin{bmatrix} 0 & 4 \\ -8 & -2 \end{bmatrix}$$

- c) Use Krasovskii's theorem to find a Lyapunov function candidate for the system below:

[10]

$$\begin{aligned} \dot{x}_1 &= -8x_1 + 3x_2 \\ \dot{x}_2 &= -3x_1 - 4x_2 - 2x_2^3 \end{aligned}$$

### Question 3

- a) Discuss 3 advantages of implementing digital control systems over analogue systems.

[6]

- b) For the system of Figure 1, find the sampled-data transfer function  $G(z) = \frac{Y(z)}{R(z)}$  if the sampling time,  $T = 0.5s$ .

[10]

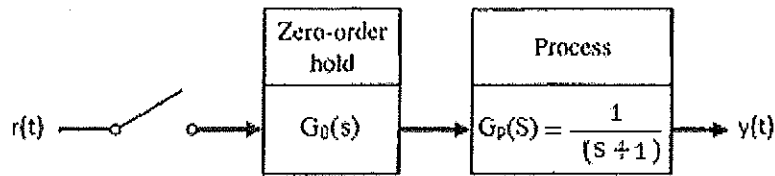


Figure 1

c) For the given system below, find the steady-state error for a ramp and parabolic input.

[9]

$$G(z) = 20 \left( \frac{T}{z-1} - 1 + \frac{z-1}{z-e^{-T}} \right)$$

#### Question 4

Determine the stability of the following systems.

i)

$$G(s) = \frac{6s + 4}{2s^2 + 6s + 4}$$

[5]

ii)

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \mathbf{u}(t)$$

$$\mathbf{y}(t) = [1 \quad 1] \mathbf{x}(t)$$

[5]

iii)

$$\dot{x} = 9x - x^3$$

(hint: find the equilibrium points first)

[7]

iv)

$$G(s) = \frac{1 - e^{-st}}{s} \frac{1}{s+2}$$

Sampling time,  $T = 0.001s$

[8]

APPENDIX

**z-Transforms**

$x(t)$	$X(s)$	$X(z)$
$u(t)$ , unit step	$1/s$	$\frac{z}{z-1}$
$t$	$1/s^2$	$\frac{Tz}{(z-1)^2}$
$e^{-at}$	$\frac{1}{s+a}$	$\frac{z}{z-e^{-aT}}$
$1 - e^{-at}$	$\frac{1}{s(s+a)}$	$\frac{(1 - e^{-aT})z}{(z-1)(z - e^{-aT})}$
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$	$\frac{z \sin(\omega T)}{z^2 - 2z \cos(\omega T) + 1}$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$	$\frac{z(z - \cos(\omega T))}{z^2 - 2z \cos(\omega T) + 1}$
$e^{-at} \sin(\omega t)$	$\frac{\omega}{(s+a)^2 + \omega^2}$	$\frac{(ze^{-aT} \sin(\omega T))}{z^2 - 2ze^{-aT} \cos(\omega T) + e^{-2aT}}$
$e^{-at} \cos(\omega t)$	$\frac{s+a}{(s+a)^2 + \omega^2}$	$\frac{z^2 - ze^{-aT} \cos(\omega T)}{z^2 - 2ze^{-aT} \cos(\omega T) + e^{-2aT}}$

Final Value Theorem for discrete signals:

$$e(\infty) = \lim_{z \rightarrow 1} 1 - z^{-1} \frac{R(z)}{1 + G(z)}$$