**University of Eswatini**
**Faculty of Science and Engineering**
**Department of Electrical and Electronic Engineering**

**Resit Examination 2021**

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| **Title of Paper:** | Modern Control Engineering |
| **Course Number:** | EEE533 |
| **Time Allowed:** | 3 hrs |

**Instructions:**

1. Answer all four (4) questions.
2. Each question carries 25 marks.

**This paper should not be opened until permission has been given by the invigilator.**

**This paper contains four (4) pages including this page.**

**Question 1**

a) For the given system, calculate the closed-loop poles required for the system response to have an overshoot of 16.32% and a settling time of 4 seconds. [10]

$$\frac{Y(s)}{U(s)} = \frac{(s+15)}{s(s+3)(s+12)}$$

b) Design the feedback gains to give the required response. [7]

c) Explain what is meant by the following terms for LTI systems.
   i) Controllability
   ii) Observability

[8]

**Question 2**

a) Determine if the following system is:
   i) Stable
   ii) Controllable

[5]

$$\dot{x}(t) = \begin{bmatrix} 0 & 3 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t)$$

b) For the same system (a), design the controller $k = R^{-1}B^TP$ for which the cost function $J = \int_0^\infty (x^TQx + u^TRu)\ dt$ is minimized, subject to $\dot{x} = Ax + Bu$ [10]

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad R = 1$$

c) List one advantage and one disadvantage of using an optimization method instead of the pole placement approach to design the full state feedback controller $u = -Kx$ [5]

**Question 3**

a) Discuss four differences between linear and non-linear systems. [8]

b) Use Lyapunov's indirect method to determine the stability of the following system, for the equilibrium point $x = 0$. [8]

$$\dot{x}_1 = -2x_2 - 4x_1(x_1^2 + x_2^3)$$

$$\dot{x}_2 = 3x_2 + x_2(x_1^2 + x_2^2)$$

c)

i) Determine all the equilibrium points for the following system [6]

$$\dot{x} = ax - x^3$$

ii) For what value of '$a$' is the origin locally asymptotically stable? [3]

**Question 4**

a) For $T = 0.2$s and $K = 10$, determine if the system on Figure 1 is stable. [15]

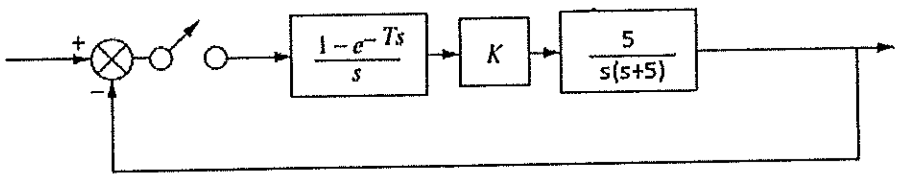


Figure 1

b) For the system on figure 2, for T = 1s, find the steady state error for a step input. [10]

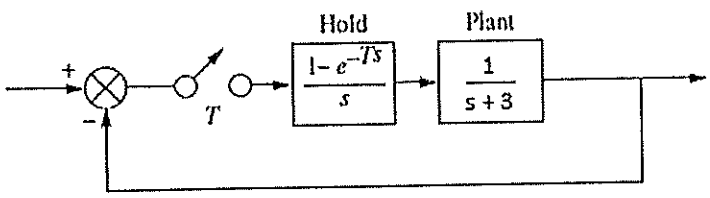


Figure 2

APPENDIX

## $z$-Transforms

| $x(t)$ | $X(s)$ | $X(z)$ |
|---|---|---|
| $u(t)$, unit step | $1/s$ | $\dfrac{z}{z-1}$ |
| $t$ | $1/s^2$ | $\dfrac{Tz}{(z-1)^2}$ |
| $e^{-at}$ | $\dfrac{1}{s+a}$ | $\dfrac{z}{z-e^{-aT}}$ |
| $1-e^{-at}$ | $\dfrac{1}{s(s+a)}$ | $\dfrac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$ |
| $\sin(\omega t)$ | $\dfrac{\omega}{s^2+\omega^2}$ | $\dfrac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$ |
| $\cos(\omega t)$ | $\dfrac{s}{s^2+\omega^2}$ | $\dfrac{z(z-\cos(\omega T))}{z^2-2z\cos(\omega T)+1}$ |
| $e^{-at}\sin(\omega t)$ | $\dfrac{\omega}{(s+a)^2+\omega^2}$ | $\dfrac{(ze^{-aT}\sin(\omega T))}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$ |
| $e^{-at}\cos(\omega t)$ | $\dfrac{s+a}{(s+a)^2+\omega^2}$ | $\dfrac{z^2-ze^{-aT}\cos(\omega T)}{z^2-2ze^{-aT}\cos(\omega T)+e^{-2aT}}$ |

Final Value Theorem for discrete signals:

$$e(\infty) = \lim_{z\to 1} 1 - z^{-1}\frac{R(z)}{1+G(z)}$$