UNIVERSITY OF SWAZILAND

Final Examination 2005

Title of Paper : Algebra, Trigonometry & Analytic Geometry

Program : BSc./B.Ed. I

Course Number : M 111

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of SEVEN questions on FOUR pages.
2. Answer any five (5) questions.
3. Non-programmable calculators may be used.

Special Requirements: None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

Question 1

(a) Use the long division to find the quotient and the remainder when $(6x^3 - 5x^2 + x - 4)$ is divided by $(2x^2 - x + 3)$.

[6 marks]

(b) Use synthetic division to find the quotient and the remainder when $(x^4 + 8x + 2)$ is divided by $(x + 1)$.

[6 marks]

(c) Find the three distinct cube roots of 1 leaving your answers in the form $a + bi$.

[8 marks]

Question 2

(a) Draw the circle $x^2 + y^2 + 6x - 2y + 20 = 0$, indicating its centre and radius.

[6 marks]

(b) For the ellipse

$$x^2 + 16y^2 + 96y + 128 = 0$$

find the centre, vertices, foci and directrices. Sketch the curve.

[7 marks]

(c) For the hyperbola

$$4x^2 - 3y^2 - 16x - 18y + 1 = 0$$

find the centre, vertices, foci, directrices and asymptotes. Sketch the curve.

[7 marks]

Question 3

(a) Find $\sin\theta$ and $\cos\theta$ if $(-8, -15)$ is on the terminal side of θ.

[6 marks]

(b) If $\sin x = \frac{-\sqrt{5}}{5}$ and $\cos x = \frac{2\sqrt{5}}{5}$, find $\cos 2x$, $\sin 2x$ and $\tan 2x$.

[8 marks]

(c) Prove that

$$\frac{2\tan\theta}{1+\tan^2\theta} = 2\sin\theta\cos\theta$$

[6 marks]

Question 4

(a) Evaluate the following

(i) $(-1-i)(-2+3i)$

(ii) $\frac{2+3i}{-1+i}$

[6 marks]

(b) Write the first four terms of

$$\frac{1}{(1-2x)^2}$$

[8 marks]

and simplify, using the binomial expansion.

(c) The third term of an arithmetic progression is 5 and the 8^{th} is 15. Find
(i) the 30^{th} term

(ii) the sum of the first 30 terms.

[6 marks]

Question 5

(a) (i) Find the equation of the line through $(-1, -1)$ and parallel to the line $x + y = 1$.

[5 marks]

(ii) Find the equation of the line through $(0, -4)$ that is perpendicular to the line $2y + x + 3 = 0$.

[5 marks]

(b) Use the mathematical induction to prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

[10 marks]

Question 6

(a) Prove the following identities

(i) $\sin^4\theta - cos^4\theta + \dfrac{2\cot^2\theta}{\csc^2\theta} = 1$

[6 marks]

(ii) $\dfrac{\cos\theta}{1 - \tan\theta} + \dfrac{\sin\theta}{1 - \cot\theta} = \sin\theta + \cos\theta$

[6 marks]

(b) A hyperbola has foci at $(0, 0)$ and $(0, 4)$. The hyperbola passes through the point $(12, 9)$. Find the equation of the hyperbola.

[8 marks]

Question 7

(a) (i) Define an ellipse.

(ii) Find the centre, the foci and the end points of the major and minor axes for the ellipse

$$\frac{x^2}{25} + \frac{y^2}{169} = 1$$

[7 marks]

(b) Use the mathematical induction to prove that

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

[8 marks]

(c) Compute $(1+i)^{20}$ using DeMoivre's theorem and leave your answer in the form $a + bi$.

[5 marks]

******** *END OF EXAMINATION* ********