6

UNIVERSITY OF SWAZILAND

Supplementary Examination 2005

Title of Paper : Algebra, Trigonometry and Analytic Geometry

Program : BSc./B.Ed. I

Course Number : M 111

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of SEVEN questions on THREE pages.
2. Answer any five (5) questions.
3. Non-programmable calculators may be used.

Special Requirements: None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

Question 1

(a) Use mathematical induction to prove that

$$3 + 5 + 7 + \cdots + (2n + 1) = n(n + 2)$$

[12 marks]

(b) Given the circle

$$2x^2 + 2y^2 + 2x - 6y - 45 = 0,$$

find:

(i) the centre;

[4 marks]

(ii) the radius.

[4 marks]

Question 2

(a) Express the following complex numbers in the form $a + ib$

(i) $\dfrac{3 - 2i}{2 + 5i}$,

[6 marks]

(ii) $(\cos 15^o + i \sin 15^o)^{12}$.

[6 marks]

(b) Find the first four terms in the expansion

$$(1 - x^3)^{\frac{1}{4}}.$$

[8 marks]

Question 3

(a) Consider the following system of linear equations

$$\begin{aligned} x + y - 2z &= 5 \\ 2x - y + z &= 2 \\ x - 2y - z &= 1 \end{aligned}$$

OR

$$\begin{pmatrix} 1 & 1 & -2 \\ 2 & -1 & 1 \\ 1 & -2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = AX = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}, \qquad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, A = \begin{pmatrix} 1 & 1 & -2 \\ 2 & -1 & 1 \\ 1 & -2 & -1 \end{pmatrix}.$$

Solve the system in three ways as follows:

(a) using Gaussian elimination;

[6 marks]

(b) using Cramer's rule;

[6 marks]

(c) using the inverse of the matrix A.

[8 marks]

Question 4

(a) Find the quotient and remainder when $4x^5 + 9x^3 - 3x^2 - 2x + 4$ is divided by $x - 1$.

[4 marks]

(b) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 4 \\ -2 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 4 & 5 \\ -2 & -1 & 0 \end{pmatrix}.$

Compute, where possible

(i) AB (ii) $A + 3B$ (iii) BA^T (iv) A^TC (v) AC^T

Indicate clearly anyone of (i)-(v) which does not exist.

[16 marks]

Question 5

(a) Given that $z = -1 + i$ is a root of the equation

$$z^4 - 2z^3 - z^2 + 2z + 10 = 0,$$

find the remaining roots. **[10 marks]**

(b) Find the four distinct 4th roots of the complex number

$$z = -8 - 8\sqrt{3}\,i.$$

[10 marks]

Question 6

(a) Given the points $A(-3, 2)$ and $B(5, 6)$, find the equation of the straight line which passes through the midpoint of AB and which is perpendicular to the line $y + 2x = 2$. **[6 marks]**

(b) Find the centre, vertices, foci eccentricity and directrices. Sketch the curve.

$$3x^2 + 4y^2 - 16y - 92 = 0$$

[14 marks]

Question 7

(a) Solve the following equation for x, where x is in the range $0 \leq x < 360^o$

$$4 - 5\sin x - 2\cos^2 x = 0.$$

[8 marks]

(b) Given that $\sin A = \frac{3}{5}$ and $\cos B = \frac{-5}{13}$, where A is in QI and B is in QII, find

(i) $\sin(A - B)$,

(ii) $\cos(A + B)$.

[12 marks]

******** *END OF EXAMINATION* ********