UNIVERSITY OF SWAZILAND

Final Examination 2005

Title of Paper	:	Introduction to Calculus
Program	:	BSc./B.Ed./B.A.S.S. I
Course Number	:	M 115
Time Allowed	:	Three (3) Hours
Instructions	:	1. This paper consists of SEVEN questions on FOUR pages. 2. Answer any five (5) questions. 3. Non-programmable calculators may be used.

Special Requirements: None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

Question 1

(a) Evaluate the following first derivatives $\dfrac{dy}{dx}$

    (i) $y = 3x^4 - x^2 + 2x$

    (ii) $\begin{cases} x &= 3t^2 - 26 \\ y &= t^3 - 3t \end{cases}$

    (iii) $y = x^{\sin x}$

                                                    [12 marks]

(b) Evaluate the second derivatives $\dfrac{d^2y}{dx^2}$ of the following

    (i) $y = \sin(3x + 2)$

    (ii) $xy + y^2 = 1$

                                                    [8 marks]

Question 2

(a) Evaluate the following indefinite integrals

    (i) $\displaystyle \int \left(x^4 + 3x^2 + \frac{1}{x} + \frac{1}{x^5} \right) \, dx$

    (ii) $\displaystyle \int \cos^3 x \, dx$

    (iii) $\displaystyle \int \frac{1}{x\sqrt{\ln x}} \, dx$

                                                    [12 marks]

(b) Derive the reduction formula in (i) and use it to evaluate the integral in (ii)

    (i) $\displaystyle \int \sin^m x \, dx = \frac{-\sin^{m-1} x \cos x}{m} + \frac{m-1}{m} \int \sin^{m-2} x \, dx$

    (ii) $\displaystyle \int \sin^5 x \, dx$ **[8 marks]**

Question 3

(a) Evaluate the following limits

(i) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x - 2}$

(ii) $\lim\limits_{x \to 0} \dfrac{\sqrt{1 - x} - \sqrt{1 + x}}{x}$

[8 marks]

(b) Use the definition (not formulas) to find the derivatives of the following functions

(i) $f(x) = 3x^2 + 5$

(ii) $f(x) = \sqrt{x}$

(iii) $f(x) = x^4$

[12 marks]

Question 4

(a) Find the equation of the tangent to the curve

$$y = x^4 - 2x^3 + 3$$

at the point $(-1, 6)$.

[8 marks]

(b) Use the substitution $u = \tan\left(\dfrac{x}{2}\right)$ to evaluate

$$\int \frac{1}{1 + \sin x - \cos x}$$

[4 marks]

(c) Find the area enclosed between $y = 10 + 3x - x^2$ and $y = 2x + 4$.

[8 marks]

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Question 5

(a) If n is a positive integer, make deductions about the nth derivative $\dfrac{d^n y}{dx^n}$ for

$$y = \frac{1}{2 - 3x}.$$

[6 marks]

(b) Use trig substitution to evaluate the following

(i) $\displaystyle \int \frac{dx}{x\sqrt{x^2 - 1}}$ [5 marks]

(ii) $\displaystyle \int \frac{x^2}{(9 - x^2)^{\frac{3}{2}}}\, dx$ [5 marks]

(c) Evaluate the following definite integral

$$\int_0^2 (16x - 3x^2 + x^3)\, dx.$$

[4 marks]

Question 6

(a) Show that

$$\int \sqrt{1 + x}\, dx = \frac{2}{15}(1 + x)^{\frac{3}{2}}(3x - 2) + c$$

in two ways

(i)   Using the substitution $u = \sqrt{1 + x}$;   [5 marks]

(ii)   Using integration by parts.   [5 marks]

(b) Find $\dfrac{dy}{dx}$ in the following

$$y \cos 2x = x \sin 2y.$$

[5 marks]

(c) Evaluate $\dfrac{d^2 y}{dx^2}$ if

$$y = \cosh(3x^2 + 5).$$

[5 marks]

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Question 7

(a) Use partial fractions to evaluate the following integrals

(i) $\int \dfrac{x}{x^2 + 2x - 3}\, dx$

[5 marks]

(ii) $\int \dfrac{x^4 - x^3 - x - 1}{x^3 - x^2}\, dx$

[7 marks]

(b) Find $\dfrac{dy}{dx}$ in the following

(i) $y = \arccos\left(\frac{1}{x}\right)$

[4 marks]

(ii) $y^2 = \dfrac{x}{x+1}$

[4 marks]

******** *END OF EXAMINATION* ********