

# UNIVERSITY OF SWAZILAND



## Supplementary Examination 2005

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- Title of Paper** : Introduction to Calculus
- Program** : BSc./B.Ed./B.A.S.S. I
- Course Number** : M 115
- Time Allowed** : Three (3) Hours
- Instructions** :
1. This paper consists of SEVEN questions on FOUR pages.
  2. Answer any five (5) questions.
  3. Non-programmable calculators may be used.
- Special Requirements:** None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

Question 1

(a) Evaluate the following

(i)  $\int_0^1 (x^2 + 1)^{11} x dx$

[5 marks]

(ii)  $\int x^2 \sin x dx$

[5 marks]

(b) Determine the value of  $k$  given that the tangent to the curve

$$y = x^2 + kx + 1$$

at the point  $(0, 1)$  is parallel to the straight line  $y - 2x - 4 = 0$ .

[5 marks]

(c) Use implicit differentiation to find  $\frac{dy}{dx}$  given that

$$\cos(xy) + x^2y + y = 0.$$

[5 marks]

Question 2

(a) Find the area bounded by the curve  $y = x^2 + 1$  and the straight line  $y = 3x - 1$ .  
[10 marks]

(b) One side of a rectangular field is bounded by a straight river. The other three sides are bounded by straight fences. The total length of the fences is 800 metres. Determine the dimensions of the rectangular field given that its area is a maximum.  
[10 marks]

Question 3

(a) Use logarithmic differentiation to find  $\frac{dy}{dx}$ :

(i)  $y = x^x$

[5 marks]

(ii)  $y = x^{\sin x}$ .

[5 marks]

(b) Derive a reduction formula for

$$\int \sec^n x \, dx,$$

[4 marks]

and hence evaluate

$$\int \sec^3 x \, dx.$$

[5 marks]

Question 4

(a) Evaluate the following limits

(i)  $\lim_{x \rightarrow 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}}$

[5 marks]

(ii)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{4x^3 - x - 3}$

[5 marks]

(b) Use the definition (not formulas) to compute the derivatives for the following

(i)  $f(x) = x^3 - 3x$

[5 marks]

(ii)  $f(x) = \frac{x-1}{x+2}$

[5 marks]

Question 5

(a) Find the area bounded by the curve  $y = 4x - x^2$  and the straight line  $y = 3x$ .  
[7 marks]

(b) Use partial fractions to evaluate

$$\int \frac{8x^2 + 8x + 8}{x^3 - x^2 - x + 1} dx.$$

[7 marks]

(c) Evaluate the following integral

$$\int \frac{dx}{(4 + x^2)^2}.$$

[6 marks]

Question 6

(a) Find  $\frac{dy}{dx}$  for the following, in as simplified a form as possible:

(i)  $y = (x^2 + 1)^{\frac{3}{2}} + \sqrt{x^2 + 1}$

[5 marks]

(ii)  $x + y^5 + x^2 + y = 0$

[5 marks]

(b) Evaluate the following integrals

(i)  $\int \frac{x}{(x-1)^2(x+1)} dx$

[5 marks]

(ii)  $\int x(\ln x)^2 dx$

[5 marks]

Question 7

(a) Use Leibnitz's rule to find  $f^{(4)}(x)$  for  $y = x^6 \ln x$ .

[5 marks]

(b) Find  $\frac{dy}{dx}$  for  $y = x^2 \arcsin(\frac{1}{x})$ .

[5 marks]

(c) Evaluate

(i)  $\int \cos^3 x \sin^2 x \, dx$

[5 marks]

(ii)  $\int \frac{x^2}{(9-x^2)^{\frac{3}{2}}} \, dx$

[5 marks]

\*\*\*\*\* END OF EXAMINATION \*\*\*\*\*