



University of Swaziland

Final Examination 2004/2005

B.Sc./B.Ed./B.A.S.S. III

Title of Paper : **Calculus I**

Course Number : **M 211**

Time Allowed : **Three (3) hours**

Instructions :

1. This paper consists of **seven questions**.
2. Answer **any five questions**.
3. Your work must be accompanied by appropriate explanations.
4. Use of **cellular phones** during the examination is not allowed.
5. Only non-programmable calculators may be used.

Special requirements: None

The examination paper must not be opened until permission has been granted by the Invigilator.

Q1.

Find the extrema of :

1. $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.

10 [marks]

2. $f(x) = 2x - 3x^{\frac{2}{3}}$ on the interval $[-1, 3]$.

10 [marks]

Q2.

(a) What is meant by an **increasing /decreasing function**?(b) Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing.

10 [marks]

(c) State the First Derivative Test theorem. Apply this theorem to find the relative extrema of the function $f(x) = \frac{1}{2}x - \sin x$ in the interval $(0, 2\pi)$.

10 [marks]

Q3.

Evaluate the following limits:

1. $\lim_{x \rightarrow \infty} \frac{3x - 2}{\sqrt{2x^2 + 2}}$

2. $\lim_{x \rightarrow -\infty} \frac{x^2}{e^{-x}}$

3. $\lim_{x \rightarrow 0^+} (1 + x)^{\frac{1}{x}}$.

20 [marks]

Q4.

(a) Use the disc method to find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2 + 1$, $y = 0$, $x = 0$ and $x = 1$ about the y -axis.

10 [marks]

(b) Repeat the same problem using the shell method. 10 [marks]

Q5.

(a) Find the arc length of the graph $(y-1)^3 = x^2$ on the interval $[0, 8]$. Hint: Solve for x in terms of y .

10 [marks]

(b) Find the arc length from $(-3, 4)$ clockwise to $(4, 3)$ along the circle $x^2 + y^2 = 25$. Show that the result is one-fourth of the circumference of the circle.

10 [marks]

Q6. Test for convergence or divergence using any appropriate test. Identify the test used.

$$1. \sum_{n=1}^{\infty} \frac{3}{n\sqrt{n}}.$$

$$2. \sum_{n=1}^{\infty} \frac{\cos n}{2^n}.$$

$$3. \sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n2^n}.$$

20 [marks]

Q7.

In the study of the progeny of rabbits, Fibonacci encountered the sequence defined recursively by: $a_{n+2} = a_n + a_{n+1}$, where $a_1 = 1$ and $a_2 = 1$.

(a) Write the first eight terms of the sequence.

(b) Write the first eight terms of the sequence defined by: $b_n = \frac{a_{n+1}}{a_n}$, for $n \geq 1$.

(c) Using (b), show that; $b_n = 1 + \frac{1}{b_{n-1}}$.

(d) The **golden ration** can be defined by $\lim_{n \rightarrow \infty} b_n = \rho$. Show that: $\rho = 1 + \frac{1}{\rho}$ and solve this equation for ρ .

20 [marks]

END OF QUESTION PAPER