



# University of Swaziland

Supplementary Examination 2004/2005

B.Sc./B.Ed./B.A.S.S. III

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**Title of Paper** : **Calculus I**

**Course Number** : M 211

**Time Allowed** : Three (3) hours

**Instructions** :

1. This paper consists of **seven questions**.
2. Answer **any five questions**.
3. Your work must be accompanied by appropriate explanations.
4. Use of **cellular phones** during the examination is not allowed.
5. Only non-programmable calculators may be used.

**Special requirements:** None

The examination paper must not be opened until permission has been granted by the Invigilator.

Q1.

- (a) Define the terms **relative maximum** and **relative minimum**.  
 (b) Find the value of the derivative at each of the given relative extrema of the following functions.

1.  $f(x) = \frac{9(x^2 - 3)}{x^3}$ , at  $(3, 2)$ .

2.  $f(x) = |x|$ , at  $(0, 0)$ .

3.  $f(x) = \sin x$ , at  $(\frac{\pi}{2}, 1)$  and at  $(\frac{3\pi}{2}, -1)$ .

20 [marks]

Q2.

- (a) Define **concavity** and state the test for concavity.  
 (b) Determine the open intervals on which the graph of  $f(x) = 6(x^2 + 3)^{-1}$  is concave upward or downward.  
 (c) State the Second Derivative Test theorem. Apply this theorem to find the relative extrema of the function  $f(x) = x^3 - 3x^2 + 3$ .

10 [marks]  
10 [marks]

Q3.

Evaluate the following limits:

1.  $\lim_{x \rightarrow \infty} \frac{2x - 1}{x + 1}$

2.  $\lim_{x \rightarrow \infty} \frac{\sin 2x}{x}$

3.  $\lim_{x \rightarrow 0^+} (\sin x)^x$ .

20 [marks]

Q4.

- (a) Use the disc or shell method to find the volume of the solid formed by revolving the region bounded by the graphs of the equations about the specified line.

$$y = x^3, y = 0, x = 2,$$

(a) the  $x$ - axis, (b) the  $y$ - axis, (c) the line  $x = 4$ , (d) the line  $y = 8$ .

20 [marks]

Q5.

(a) Find the arc length from  $(x_1, y_1)$  to  $(x_2, y_2)$  on the graph  $f(x) = mx + b$ , where  $m$  and  $b$  are real constants.

10 [marks]

(b) Find the arc length of the graph  $y = \frac{x^3}{6} + \frac{1}{2x}$  on the interval  $[\frac{1}{2}, 2]$ .

10 [marks]

Q6. Test for convergence or divergence using any appropriate test. Identify the test used.

1.  $\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$ .

2.  $\sum_{n=1}^{\infty} ne^{-n^2}$ .

3.  $\sum_{n=1}^{\infty} (-1)^n \frac{3}{4n+1}$ .

20 [marks]

Q7.

(a) Determine whether the series converges conditionally or absolutely, or diverges.

1.  $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ .

2.  $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^3 - 1}$ .

3.  $\sum_{n=0}^{\infty} \frac{\cos n\pi}{n^2}$ .

20 [marks]

END OF QUESTION PAPER