University of Swaziland

Final Examination 2004/2005

B.Sc./B.Ed./B.A.S.S. III

Title of Paper : **Calculus II**

Course Number : M 212

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of **seven questions.**
2. Answer **any five questions**.
3. Your work must be accompanied by appropriate explanations.
4. Use of **cellular phones** during the examination is not allowed.
5. Only non-programmable calculators may be used.

Special requirements: None

The examination paper must not be opened until permission has been granted by the Invigilator.

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Q1.
(a) Sketch the curve represented by the equations: $x = \sin t, y = \sin^2 t$, by eliminating the parameter and finding the corresponding rectangular equation and its domain.

8 [marks]

(b) Sketch and identify the curve defined by the parametric equations:

$$x = t^2 - 2t, y = t + 1.$$

12 [marks]

Q2.
(a) Find the length of the curve given by:

$$x = t^3, y = t^2, 0 \leq t \leq 4.$$

10 [marks]

(b) Find the length of one arch of the cycloid:

$$x = r(\theta - \sin\theta), y = r(1 - \cos\theta), 0 \leq \theta \leq 2\pi.$$

10 [marks]

Q3.
(a) Find the domains of the following functions and evaluate $f(3,2)$.

1. $f(x,y) = \dfrac{\sqrt{x+y+1}}{x-1}$.
2. $f(x,y) = x\ln(y^2 - x)$.

Sketch these domains.
(b) Sketch the level curves of the function: $g(x,y) = \sqrt{9 - x^2 - y^2}$ for $k = 0, 1, 2, 3$.

20[marks]

Q4.

(a) If $f(x,y) = \sin(\frac{x}{y+1})$, calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined implicitly as a function of x and y by the equation: $x^3 + y^3 + z^3 + 6xyz = 1$.

12 [marks]

(b) Show that the function $u(x,y) = e^x \sin y$ satisfies the equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

8 [marks]

Q5.

A rectangular box is resting on the $xy-$ plane with one vertex at the origin. The opposite vertex lies in the plane $6x + 4y + 3z = 24$. Find the maximum volume of such a box.

20 [marks]

Q6.

(a) If $f(x,y) = xe^y$, find the rate of change of f at the point $P(2,0)$ in the direction from P to $Q(5,4)$.

In what direction does f have the maximum rate of change? What is this maximum rate of change?

(b) Suppose the temperature at a point (x,y,z) in space is given by $T(x,y,z) = \frac{80}{(1+x^2+2y^2+3z^2)}$, where T is measured in degrees Celsius and x, y, z in metres. In which direction does the temperature increase fastest at the point $(1, 1-2)$? What is this maximum rate of inrease?

20 [marks]

Q7.

(a) Use an iterated integral to find the area of the region bounded by the graphs of $f(x) = \sin x$ and $g(x) = \cos x$ between $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

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(b) Find the area of the region R that lies below the parabola $y = 4x - x^2$ above the $x-$ axis, and above the line $y = -3x + 6$. Note: the line and $x-$ axis form the lower boundary.

20 [marks]

END OF QUESTION PAPER