



University of Swaziland

Supplementary Examination 2004/2005

B.Sc./B.Ed./B.A.S.S. III

Title of Paper : Calculus II

Course Number : M 212

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of **seven questions**.
2. Answer **any five questions**.
3. Your work must be accompanied by appropriate explanations.
4. Use of **cellular phones** during the examination is not allowed.
5. Only non-programmable calculators may be used.

Special requirements: None

The examination paper must not be opened until permission has been granted by the Invigilator.

Q1.

(a) Let $z = f(x, y)$. Define the **total differential** of the dependent variable z .

If $z = f(x, y) = x^2 + 3xy - y^2$, find the differential dz . If x changes from 2 to 2.05 and y changes from 3 to 2.96, compare the values of dz and Δz .

(b) Use differentials to find an approximate value for $\sqrt{9(1.95)^2 + (8.1)^2}$.

20 [marks]

Q2.

(a) A rectangular box with an open top has a length of x metres, a width of y metres, and a height of z metres. Express the cost C of constructing the box as a function of x , y and z if it costs \$0.75 per square feet to build the base and \$0.40 per square metre to build the sides. 10 [marks]

(b) A propane tank is constructed by welding hemispheres to the ends of a right circular cylinder. Write the volume V as a function of r and l where r is the radius of the cylinder and hemispheres and l is the length of the cylinder.

10 [marks]

Q3.

(a) By integrating first with respect to x , evaluate $\int \int_R y \sin(xy) dA$, where $R = [1, 2] \times [0, \pi]$.

(b) Now repeat the problem, but this time integrate with respect to y first.

20 [marks]

Q4.

Evaluate $\int \int \int_E z dV$ where E is the solid region bounded by the four planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

20 [marks]

Q5.

For the cardioid $r = 1 + \sin \theta$, find the slope of the tangent line when $\theta = \frac{\pi}{3}$.
 Find the points on the cardioid where the tangent line is horizontal or vertical.
 20 [marks]

Q6.

(a) If $z = x^2y + 3xy^4$, where $x = e^t$ and $y = \sin t$, find $\frac{dz}{dt}$.

Suppose that the pressure P , volume V and temperature T are related by the equation $PV = 8.31T$. Find the rate at which the pressure is changing when the temperature is 300 degrees Kelvin and increasing at a rate of 0.1 degrees Kelvin per second and the volume is 100L and increasing at a rate of 0.2L per second.

(b) If $z = e^x \sin y$ where $x = st^2$ and $y = s^2t$, find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$

20 [marks]

Q7.

(a) Find the equation of the tangent plane to the paraboloid: $z = 1 - \frac{1}{10}(x^2 + 4y^2)$ at the point $(1, 1, \frac{1}{2})$.
 10[marks]

(b) Find a set of symmetric equations for the normal line to the surface given by $xyz = 12$ at the point $(2, -2, -3)$.
 10[marks]

END OF QUESTION PAPER