

UNIVERSITY OF SWAZILAND**Final Examination 2005**

Title of Paper : Ordinary Differential Equations

Program : BSc./B.Ed./B.A.S.S./B.Eng. II

Course Number : M 213

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of SEVEN questions on FOUR pages.
2. Answer any five (5) questions.
3. Non-programmable calculators may be used.

Special Requirements: None

THIS EXAMINATION PAPER MAY NOT BE OPENED UNTIL PERMISSION TO DO SO IS GRANTED BY THE INVIGILATOR.

Question 1

(a) Find the general solution of

$$(2xy^4e^y + 2xy^3) dx + (x^2y^4e^y - x^2y^2 - 3x) dy = 0.$$

[8 marks]

(b) $Y' = AY$ where $A = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix}$ $Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$
 $y_1(0) = 2$ $y_2(0) = 1$

[12 marks]

Question 2

Show that the general solution of

$$\frac{dy}{dx} + p(x)y = q(x)$$

has the form

$$y = e^{-\int p(x)dx} \left[C + \int q(x)e^{\int p(x)dx} dx \right]$$

where C is a constant.

Using the above or otherwise integrate

$$x \frac{dy}{dx} - y + 3x^3y - x^2 = 0$$

[20 marks]

Question 3

(a) Use Laplace transform to solve the following initial value problem

$$y'' + 2y' + 5y = 0$$

$$y(0) = 1 \quad y'(0) = 5.$$

[12 marks]

(b) Solve

$$x^2 \frac{dy}{dx} + 5xy = \frac{\cos x}{x^3}.$$

[8 marks]

Question 4

(a) Find the series solution of the equation

$$2xy'' + (x+1)y' + y = 0$$

about $x = 0$.

[8 marks]

(b) Solve

$$\left. \begin{aligned} 2\frac{d^2x}{dt^2} + \frac{dy}{dt} + x &= 2 \\ \frac{dx}{dt} + \frac{d^2y}{dt^2} + 2y &= 2t \end{aligned} \right\} \begin{aligned} x(0) = y(0) &= 0 \\ x'(0) = y'(0) &= 0 \end{aligned}$$

[12 marks]

Question 5

(a) Given that $y_1(x) = e^{2x} \cos x$ and $y_2(x) = e^{2x} \sin x$ are solutions of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$$

find that solution which satisfies the conditions $y\left(\frac{\pi}{4}\right) = 1$, $y\left(\frac{\pi}{3}\right) = 2$.

[5 marks]

(b) Find a general solution for

$$y''' - y = xe^x.$$

[5 marks]

(c) Find particular solutions of the differential equations

(i) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 4y = x^2 + 4$

[5 marks]

(ii) $\frac{d^3y}{dx^3} + 2\frac{dy}{dx} = x^2 - x$

[5 marks]

Question 6

Find the general solutions

(a) $xy \, dy + (1 + y^2) \, dx = 0$

[6 marks]

(b) $(x^2 - y) \, dx + (y^2 - x) \, dy = 0$

[7 marks]

(c) $(x^2 - xy + y^2) \, dx - xy \, dy = 0$

[7 marks]

Question 7

(a) Find the series solution of the equation

$$2xy'' + (x + 1)y' + 3y = 0$$

about $x = 0$.

[10 marks]

(b) Solve

$$\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3}$$

[10 marks]

***** END OF EXAMINATION *****