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UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2005

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE NUMBER	:	M 213
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) If $F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$ is the Laplace transform of $f(t)$, show that

   i. $\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$ [6 marks]

   ii. $\mathcal{L}\{f''(t)\} = s^2\mathcal{L}\{f(t)\} - sf'(0) - f(0)$ [6 marks]

   (b) Use the method of variation of parameters to solve

$$y'' + y = \tan x$$

[8 marks]

QUESTION 2

2. (a) Use the method of undertemined coefficients to solve

$$y'' + 4y' + 3y = 5\sin 2x$$

[8 marks]

   (b) Obtain the series solution of

$$2xy'' + (1+x)y' + y = 0$$

   about $x = 0$ [12 marks]

QUESTION 3

3. (a) Use the method of separation of variables to solve

   i. $e^y y' - x - x^3 = 0$ [5 marks]

   ii. $yy' + (1 + y^2)\sin x = 0$ [7 marks]

   (b) Solve

$$\begin{aligned} y_1' + y_2 &= 0 \\ y_2' - y_1 &= 0 \\ y_1(0) = 1 \qquad & y_2(0) = 3 \end{aligned}$$

[8 marks]

QUESTION 4

4. (a) Integrate

$$y' = Ay \qquad \text{where} \qquad A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \qquad Y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

[10 marks]

   (b) Show that the differential equation

$$(3x + \cos y)dy + (3y + e^x)dx = 0$$

   is exact and find the solution. [10 marks]

QUESTION 5

5. (a) Obtain an integrating factor and solve the differential equation

$$\left(\frac{y^2}{2} + 2ye^x\right) dx + (y + e^x)dy$$

[10 marks]

(b) Prove that the differential equation

$$xdy + ydx - 2x^2y^3dy = 0$$

has an integrating factor of the form $x^n y^n$ and solve the equation [10 marks]

QUESTION 6

6. Use the method of Frobenius to obtain two linearly independent solutions of the differential equation

$$2x^2y'' + (x^2 - x)y' + y = 0$$

about $x = 0$.

Give each solution in the form of a suitable series with only the first five terms. [20 marks]

H2

QUESTION 7

7. Use Laplace Transform technique to solve

(a)

$$y'' - 3y' + 2y = 6e^{-x}, \quad y(0) = 3, \quad y'(0) = 3$$

[8 marks]

(b)

$$y'' + 2y' + 5y = 0$$

[6 marks]

(c)

$$y'' - y = xe^{x}$$

[6 marks]

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathscr{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathscr{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}\,t^{n-1}$$

$$\mathscr{L}[e^{at}] = \frac{1}{s-a} \qquad \mathscr{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathscr{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathscr{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathscr{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathscr{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathscr{L}[D^n x] = s^n\mathscr{L}[x] - s^{n-1}x(0) - s^{n-2}x'(0) - \cdots - x^{(n-1)}(0)$$

$$\mathscr{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\,\mathscr{L}[f(t)] \qquad \mathscr{L}^{-1}\left[\frac{1}{s}\,F(s)\right] = \int_0^t \mathscr{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathscr{L}[f(t)]$, so $f(t) = \mathscr{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathscr{L}[e^{at}f(t)] = F(s-a) \qquad \mathscr{L}^{-1}[F(s)] = e^{at}\mathscr{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathscr{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\,\mathscr{L}[f(t)] \qquad \mathscr{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathscr{L}[u_a(t)g(t)] = e^{-as}\mathscr{L}[g(t+a)] \qquad \mathscr{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

Convolution

$$\mathscr{L}^{-1}[F(s)G(s)] = \mathscr{L}^{-1}[F(s)] * \mathscr{L}^{-1}[G(s)]$$

where

$$(f * g)(t) = \int_0^t f(t-u)g(u)\,du.$$

Periodic Functions

If $f(t+p) = f(t)$ for all t, then

$$\mathscr{L}[f(t)] = \frac{\displaystyle\int_0^p e^{-st}f(t)\,dt}{1-e^{-ps}}.$$