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UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2005

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Find the values of b and c for which the vectors $[2, -3, 4]$ and $[1, b, c]$ are parallel.

(b) Find the values of λ for which the vectors $[\lambda, -2, 1]$ and $[2\lambda, \lambda, -4]$ are perpendicular.

(c) Confirm that the vectors $[3, 1, -2]$, $[-1, 3, 4]$, and $[4, -2, -6]$ form the sides of a triangle. [20]

QUESTION 2

(a) Use triple integration to find the volume between the spheres $x^2 + y^2 + z^2 = 16$ and $x^2 + y^2 + z^2 = 9$. [10]

(b) Use the Gaussian Elimination method or the Gauss-Jordan Elimination method to solve the following system of linear equations

$$\begin{aligned} 2x_1 - 3x_2 + 4x_3 - x_4 &= 0 \\ 7x_1 + x_2 - 8x_3 + 9x_4 &= 0 \\ 2x_1 + 8x_2 - x_3 - x_4 &= 0. \end{aligned}$$

[10]

QUESTION 3

(a) Let A and B be two points and $\vec{a}$, $\vec{b}$ be their position vectors. Show that the position vector $\vec{r}$ of the point R which divides AB in the ration $p : q$ is given by

$$\vec{r} = \frac{q\vec{a} + p\vec{b}}{p + q}.$$

Deduce the mid-point formula. [5]

(b) Evaluate the following limits

(i) $\lim_{x\to\infty} \frac{e^{\frac{3}{x}}-1}{\sin(\frac{1}{x})}$

(ii) $\lim_{x\to\infty} x^{\frac{1}{x}}$

(iii) $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\ln(1+x)}\right)$. [15]

QUESTION 4

(a) The temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 cm centered at the plate's origin. Use the method of Lagrange Multipliers to find the highest and lowest temperatures encountered by the ant. [10]

(b) Find the first four nonzero terms of the Taylor series generated by $f(x) = e^x$ about $x = 0$. Use the series to find an approximation of

$$\int_0^1 e^{-x^2} dx$$

correct to three decimal places. [10]

QUESTION 5

(a) (i) State the Mean Value Theorem.

(ii) Show that for the function $f(x) = \frac{4}{x}$ there is no real number c in the interval $(-1, 4)$ such that $f(4) - f(-1) = f'(c)[4 - (-1)]$. Why does this not contradict the Mean Value Theorem? [8]

(b) The transformation equations from rectangular coordinates (x, y, z) to cylindrical coordinates (r, θ, z) is given by

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z.$$

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Show that

$$\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = r.$$

Use this transformation to evaluate

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} (2-2x^2-2y^2)\mathrm{d}z\mathrm{d}y\mathrm{d}x.$$

[12]

QUESTION 6

(a) Let a_{11}, a_{12}, a_{21} and a_{22} be given real numbers such $a_{11}a_{22} - a_{12}a_{21} \neq 0$. Find numbers b_{11}, b_{12}, b_{21} and b_{22} such that

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

[10]

(b) Solve the linear system

$$\begin{aligned} 3x - 2y + z &= 1 \\ 3x + 3x &= 3 \\ x + y - z &= 0, \end{aligned}$$

using Cramer's rule. [10]

QUESTION 7

(a) Find the value of $\frac{\partial f}{\partial x}$ at the point $(4,5)$ if

$$f(x,y) = x^2 + 3xy + y - 1.$$

[4]

(b) If $w = \frac{x^3+y^3}{x-y}$, show that $xw_y + yw_x = \frac{\left(x+y\right)^3}{x-y}$. [10]

(c) Find conditions on a, b and c such that the system

$$\begin{aligned} ax_1 + bx_2 &= c \\ bx_1 + ax_2 &= c \end{aligned}$$

has infinitely many solutions. [6]

END OF EXAMINATION