UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2005

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Determine k so that the vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, where

(i) $\mathbf{u} = [1, k, -3]$ and $\mathbf{v} = [2, -5, 4]$

(ii) $\mathbf{u} = [2, 3k, -4, 1, 5]$ and $\mathbf{v} = [6, -1, 3, 7, 2k]$ [4]

(b) Show that the vectors $[3, -2, 1]$, $[1, -3, 5]$, and $[2, 1, -4]$ are sides of a right angled triangle. [6]

(c) If $\mathbf{a} = [3, -1, 2]$, $\mathbf{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{c} = [1, -2, 2]$, $\mathbf{u} = \mathbf{a} \times \mathbf{b}$, $\mathbf{v} = \mathbf{b} \times \mathbf{c}$, and θ is the angle between $\mathbf{u}$ and $\mathbf{v}$,

(i) find $\mathbf{u}$ and $\mathbf{v}$,

(ii) find $\cos\theta$ and $\sin\theta$,

(iii) confirm that $\sin^2\theta + \cos^2\theta = 1$. [10]

QUESTION 2

(a) Find the volume of the region in the first octant bounded by $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 2$, where a, b, and c are positive constants. [10]

(b) Use the Gaussian Elimination method or the Gauss-Jordan Elimination method to solve the following system of linear equations

$$\begin{aligned} 2x_1 - 3x_2 + 4x_3 - x_4 &= 0 \\ 7x_1 + x_2 - 8x_3 + 9x_4 &= 0 \\ 2x_1 + 8x_2 - x_3 - x_4 &= 0. \end{aligned}$$

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QUESTION 3

(a) The position vectors of the points A and B are given by $\vec{a}$ and $\vec{b}$, respectively. P and Q are two points on AB with position vectors $\vec{p}$ and $\vec{q}$, respectively, and are such that $|\overrightarrow{AP}| = |\overrightarrow{PQ}| = |\overrightarrow{QB}|$.

Show that

(a) $\vec{p} = \dfrac{2\vec{a} + \vec{b}}{3}$

(b) $\vec{q} = \dfrac{\vec{a} + 2\vec{b}}{3}$.

[10]

(b) Evaluate the following limits

(i) $\lim_{x\to\infty} \frac{e^{\frac{3}{x}}-1}{\sin(\frac{1}{x})}$

(ii) $\lim_{x\to\infty} x^{\frac{1}{x}}$. [10]

QUESTION 4

(a) The temperature at a point (x, y) on a metal plate is $T(x, y) = 4x^2 - 4xy + y^2$. An ant on the plate walks around the circle of radius 5 cm centered at the plate's origin. Use the method of Lagrange Multipliers to find the highest and lowest temperatures encountered by the ant. [10]

(b) Find the first four nonzero terms of the Taylor series generated by $f(x) = \sin(x)$ about $x = 0$. Use the series to find an approximation of

$$\sin(0.1)$$

correct to four decimal places. [10]

QUESTION 5

(a) Show that the function $f(x) = \dfrac{x^2 - 1}{x + 2}$ satisfies the hypothesis of Rolle's theorem on the interval $[-1, 1]$. Find all real numbers $c \in (-1, 1)$ such that $f'(c) = 0$.[4]

(b) Determine whether the function $f(x) = x - \dfrac{1}{x}$ satisfies the hypothesis of the Mean Value Theorem on the interval $[1, 3]$ and if so, find all real numbers $c \in (1, 3)$ such that

$$f(3) - f(1) = f'(c)[3 - 1].$$

[4]

(c) The transformation equations from rectangular coordinates (x, y, z) to cylindrical coordinates (r, θ, z) is given by

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z.$$

Show that

$$\frac{\partial(x, y, z)}{\partial(r, \theta, z)} = r.$$

Use this transformation to evaluate

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{4}(2 - 2x^2 - 2y^2)\mathrm{d}z\mathrm{d}y\mathrm{d}x.$$

[12]

QUESTION 6

(a) Find the inverse of the square matrix

$$A = \begin{pmatrix} 2 & 4 & 3 \\ 0 & 1 & -1 \\ 3 & 5 & 7 \end{pmatrix}$$

[10]

(b) Solve the linear system

$$\begin{aligned} 2x + 4y + 6z &= 18 \\ 4x + 5y + 6z &= 24 \\ 3x + y - 2z &= 4, \end{aligned}$$

using Cramer's rule. [10]

QUESTION 7

(a) Find the value of $\frac{\partial f}{\partial x}$ at the point $(3, 4)$ if

$$f(x, y) = x^2 + 3xy + y - 1.$$

[4]

(b) If $w = \frac{x^3+y^3}{x-y}$, show that $xw_y + yw_x = \frac{\left(x+y\right)^3}{x-y}$. [10]

(c) Find conditions on a, b and c such that the system

$$\begin{aligned} ax_1 + bx_2 &= c \\ bx_1 + ax_2 &= c \end{aligned}$$

has infinitely many solutions. [6]

END OF EXAMINATION